

THE CHANGE IN THE ELASTICITY OF A COPPER WIRE
WITH CURRENT AND EXTERNAL HEATING.

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THE present investigation consists of determinations of Young's modulus of a specimen of copper wire at various temperatures between 20° C. and 150° C. with the object of ascertaining whether or not heating by an electric current has any effect other than that caused by the accompanying increase of temperature, and also to learn certain facts regarding the temperature effect itself, concerning which there seems to be considerable doubt.

PREVIOUS RESULTS UPON THE EFFECT OF CURRENT HEATING.

Wertheim¹ reported a decrease in Young's modulus caused by current and believed this decrease to be independent of any temperature effect, a conclusion not sufficiently justified as he assumed the heating effect of the current negligible. Edlund² made allowance for the temperature effect, and concluded that current, except for the accompanying temperature change, does not affect Young's modulus. Streintz³ made observations of the torsion modulus at room temperature and at 55.5° C. The following figures represent the percentage change of elasticity caused by the current, correction being made for the temperature effect: Brass (*a*), - 5.9; (*b*), + 12.8; copper, 0.0; silver, + 3.9; iron, + 3.1; steel, - 12.2. Mebius⁴ found the effect due to current in the case of steel, iron, brass, and silver rods to be extremely small. Miss Noyes⁵ studied steel, silver, and copper wires and found that the current heating caused a uniform decrease in Young's modulus which could be entirely accounted for as a temperature effect. Walker⁶ employed the same experimental methods as Miss Noyes, but found very irregular changes with steel, soft iron, platinum, and copper, which could not be accounted for by temperature and which differed greatly with increasing and decreasing

¹ Ann. de Chim. et de Phys., 12, 610, 1844.

² Annal. d. Phys., 129, 15, 1866; 131, 337, 1867.

³ Annal. d. Phys., 150, 368, 1873.

⁴ Oefvers. af k. Vet. Akad. Forhandl., 681, 1887; Beibl. 12, 678.

⁵ PHYS. REV., 2, 279, 1895; 3, 433, 1896.

⁶ Proc. Roy. Soc. Edin., 27, 343, 1907.

current. A survey of previous work leaves a serious doubt as to the effect of current heating, and this doubt a critical study serves to increase.

PREVIOUS RESULTS UPON THE CHANGE OF YOUNG'S MODULUS WITH TEMPERATURE.

The investigations that have been carried on to determine the change of Young's modulus with temperature are so extensive that the results with copper only can be considered. These are however representative of the results for other metals. Shakespeare¹ has found a decrease of Young's modulus for copper of 3.6 per cent. on heating from 13° C. to 100° C. Miss Noyes² made observations at various temperatures up to 150° C. and reports a uniform decrease of the modulus of 0.13 per cent. and 0.07 per cent. per degree for two samples of wire. Gray, Blyth, and Dunlop³ also were able to make observations at intermediate temperatures up to 100° C. They also report a uniform decrease, which for two samples was 0.015 per cent. and 0.04 per cent. per degree respectively. On the other hand Slotte's⁴ observations at intervals over a range of 10° C. to 70° C. upon two samples of copper wire showed a decrease of Young's modulus with increase of temperature which was not uniform but became less rapid with one sample and more rapid with the other as the temperature increased. An increasing rate of change is also indicated by the results of Wertheim⁵ at 15° C., 100° C., and at 200° C., the change being much greater for the upper interval. In this connection the work of Pisati,⁶ Kohlrausch and Loomis,⁷ and of Slotte⁸ upon the torsion modulus is of interest. They found the rate of decrease of the modulus greater as the temperature was raised. This is suggestive as it is highly probable that the general nature of the elastic changes is similar for the two moduli. These results leave no doubt that increase of temperature causes a decrease of Young's modulus, but there is no conclusive evidence as to the exact nature of the decrease.

SCOPE OF WORK.

In view of the facts which have been cited it was decided to attempt an investigation of the effect of current upon Young's modulus, accompanied by a parallel investigation of the temperature change produced by an

¹ Phil. Mag., 47, 539, 1899.

² Loc. cit.

³ Proc. Roy. Soc., 67, 180, 1900.

⁴ Acta Soc. Scien. Fennicæ, Vol. 26, 1899.

⁵ Loc. cit.

⁶ Nuovo Cimento, 4, 152, 1878; 5, 34, 135, 1879.

⁷ Annal. d. Phys., 141, 481, 1871.

⁸ Acta Soc. Scien. Fennicæ, Vol. 35, 1908.

external source of heat. Examination of the previous work led me to believe that the desideratum was not a hastily made series of observations upon a number of metals but rather thorough and extended observations upon a single metal. Consequently this report is confined to results with copper although work is under way with other metals. A review of the literature made clear that the causes of inaccurate results in the past have been chiefly a lack of mechanical perfection in the apparatus and insufficient attention to the distribution of temperature. In none of the work upon the effect of current has there been any knowledge of the differences in temperature along the wire or any effective attempt made to secure a uniform distribution. The apparent crudity of the apparatus that is here described is the result of an attempt to keep every part of the apparatus as simple as possible and to refrain from unwarranted refinements. Each source of error has been considered in relation to the others and the effectiveness of the apparatus is best judged from the consistency of the results that have been obtained.

DESCRIPTION OF THE APPARATUS.

In order to prevent convection currents and consequent non-uniformity of heating necessarily accompanying vertical suspension the wire was mounted horizontally. The mechanical features of the apparatus (Fig. 1) may be considered independently of the heating box. The wire is

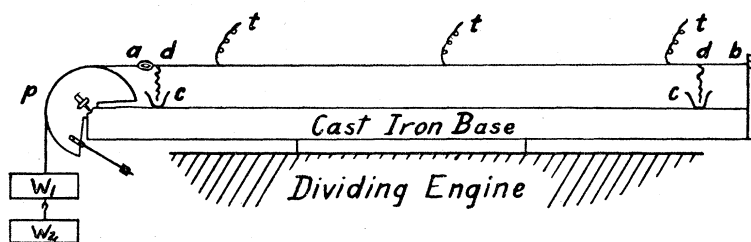


Fig. 1.

The apparatus, showing method of suspending the test specimen.

suspended between a bar (*b*) and a pulley (*p*) which are in turn supported by a heavy cast iron base, bolted to the carriage of the dividing engine. The wire is stretched by the weights W_1 and W_2 . Current is led in at the points (*d*) through the mercury cups (*c*), the wire being insulated at (*a*) and at (*b*). Three copper-constantan thermo-couples (*t*) of No. 36 wire are for temperature measurements. The weight W_1 is kept upon the wire continually and is heavy enough to stretch the wire almost straight. The weight W_2 is added gradually and without jar by a simple

apparatus operated by a foot lever. The elongation of the wire is measured with two micrometer microscopes attached to the bed of the dividing engine. These are focused upon bright scratches on the wire, made visible by incandescent lamps.

The heating apparatus (Fig. 2), although supported by the cast-iron base, is mechanically entirely independent of the wire. It must not only furnish a means of heating the wire externally but also must retain the heat when the current is passed through the wire itself. The base (*o*), bottom piece (*s*), and back (*v*) are covered with asbestos and fastened together. The asbestos-covered front (*h*), which has two windows of mica (*m*) for observing the wire, and the glass cover (*n*) are held together by clamps easily removed. The glass tubes (*k*) are large enough to give sufficient clearance to the wire (*w*) but small enough to prevent convection or other air currents. Zigzagging back and forth across the bottom piece (*s*) is a heating wire of German silver through which a current can be

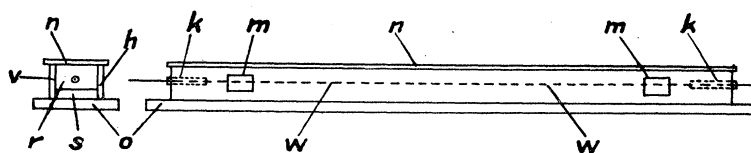


Fig. 2.

Heating box surrounding the test specimen.

sent. This heating box proved to be one of the three important features of the apparatus and its simple form in no way indicates the difficulty of securing a reasonably even temperature distribution. This fact is very suggestive in connection with the results of former investigations in which average temperature only was known, there being no information concerning the temperature distribution. The use of thermocouples which could be slipped to different portions of the wire revealed surprising inequalities of temperature. Even after uniformity of heating had been secured with the external source it was necessary to still further alter the box before a satisfactory distribution could be obtained with internal or current heating. At the highest temperature employed the maximum variation of temperature over the portion of the wire under observation was less than 15°C .

A second feature of importance is the free suspension of the wire. The possibility of error due to contact of the wire with rigid portions of the apparatus is eliminated by permitting it to touch nothing except the thermo-couples which are of fine flexible wire. Consequently the specimen hangs freely in the form of a flat catenary. The elongation

caused by the straightening of the catenary on the application of the additional stretching weight is negligible, being about 1/100 of the smallest reading of the micrometer microscope. The third feature is also mechanical. It is the pulley which changes the direction of the applied force, which must be transmitted without gain or loss resulting from friction or change of leverage. Fig. 3 shows its construction and suggests the method of centering the axis. The pulley was tested at different posi-

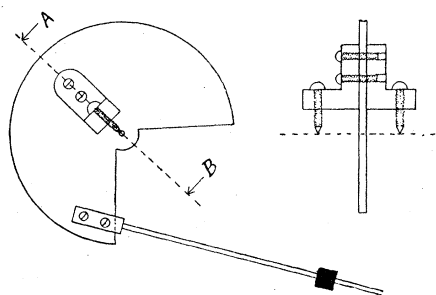


Fig. 3.

Side view of pulley and section along A-B.

tions at various times during the progress of the work and found to introduce no appreciable error.

METHOD OF DETERMINING TEMPERATURE OF SPECIMEN.

When the wire is heated by the external source it is of uniform temperature throughout its cross section and of the same temperature as the surrounding air. Under these conditions the thermo-couples which have already been mentioned are a perfectly satisfactory means of measuring its temperature. In the case of internal or current heating there is an increase of temperature toward the center, resulting in a number of effects, change of resistance, thermal conductivity, etc., each of which has its influence upon the current distribution and in turn upon the temperature distribution. The average temperature of the wire is of course a function of the surface temperature and of the current, but for evident reasons the thermo-couples do not give the surface temperature of the wire when the wire itself is the source of heat.

Measurement of temperature by change of length caused by thermal expansion has proved to be the most accurate and convenient method. Blondlot¹ has shown conclusively for several pieces of brass and one of German silver that current has no direct effect upon the dimensions of

¹ Journal de Phys., [1], 8, 122, 1879.

a metal carrying a current. This has also been shown by Righi¹ for four wires whose composition is not given. There is no reason to believe that copper should be peculiar in showing this effect. In fact our own experience and the persistency with which our results for Young's modulus at any temperature as determined by this method remain the same, no matter what may be the source of heat, is in itself proof that current has no direct effect upon the length of the wire. In using change of length as an indication of temperature account must be taken of the change caused indirectly by the effect of temperature upon Young's modulus. Thermo-couple readings and length observations were taken for all work with external heating. With internal heating length observations were made and the temperature determined by comparison with the other results. The thermo-couples were connected to a sensitive Siemens and Halske galvanometer by a three-way switch and were calibrated by direct comparison with an accurate thermometer.

With current heating the central portions of the wire tend to lengthen more than the outer portions which are at a lower temperature and it is assumed that the actual lengthening so averages this effect as to be an indication of the average temperature. It is also worthy of notice that the contribution to Young's modulus will be different for the different portions, the value observed being an average of the moduli of the various circular layers making up the cross section of the wire. There is of course a similar averaging of temperature and modulus along the length of the wire.

NATURE OF THE TESTS.

Three sections of commercial copper wire were tested, all from the same piece obtained from the Driver-Harris Wire Co., Harrison, N. J. The purity of the specimen was determined electrolytically, with results of 99.88 per cent. and 99.92 per cent. As the second was the more accurate we may take $99.91 \pm .01$ as the purity of the specimen. There was no trace of either silver, lead, or iron. The coefficient of linear expansion was .0000169. Specimen No. 2 was 0.81 mm. in diameter. The portion under observation was 50.7 cm. long. The other samples were of similar dimensions. A weight of 2,109 grams was kept upon the wires continually. The modulus was determined by measuring the elongation produced by an additional weight of 2,252 grams. The elongation was observed with a microscope, the micrometer head of which can be read to 1/50 revolution, corresponding to .00093 mm. stretch. At room temperature the stretch amounted to about .02 mm.

¹ Nuovo Cimento, [3], 7, 116, 1880.

Currents as high as 15 amperes were used. The accuracy of the work is of a high degree. The error in the determination of Young's modulus is about 1 per cent. A greater accuracy may prove desirable if small peculiar effects appear with other metals, but the present apparatus is sufficiently accurate to determine the general nature of the changes in the modulus.

Each determination of Young's modulus as represented by a point on the curves or a value in the tables is the result of ten or more separate measurements taken so as to eliminate any error caused by a slight drift in temperature. Generally a half hour or more was required for the temperature to reach a sufficiently steady state. After the preliminary work the greater part of the observations consisted of series of determinations with increasing or decreasing temperature over a range of 20° C. to 150° C.

PRELIMINARY TESTS.

The possible ways in which current may affect elasticity are so numerous that a determination of Young's modulus at a given temperature is of no value unless interpreted in connection with the whole thermal history of the wire. Since the essential part of this investigation is the comparison of observations taken under as nearly similar thermal conditions as possible, except for the difference in time and in the source of heat, an elimination as far as possible of any other than pure temperature effects is desirable. If this is impossible a thorough understanding of the nature of the other effects is essential. In order to eliminate the well-known permanent changes accompanying extreme temperatures the treatment of the wire after its first heating and stretching was confined to a temperature range of 20° C. to 150° C. In order to find out how consistent was its behavior within this range and under the various conditions of treatment a great deal of preliminary work was necessary. This showed that there are no sudden changes in the elastic state of the wire, that the rapidity of heating and cooling had little if any effect upon the changes in elasticity, and that the length of time the wire was held at a given temperature had no great effect. As explained later, permanent or history effects were found, but for any series of readings these changes were insignificant, the value of Young's modulus returning at the close of any day's observations to practically its original value.

That history effects could be eliminated and the elasticity of the wire made independent of treatment and a function of temperature only is illustrated in Fig. 4, Series 5. Observation 15 is almost identical with 8, and 16 and 17 are nearly the same as the first observations, although the wire had meantime been heated and cooled twice.

In order to learn whether or not at any given temperature the value of Young's modulus is dependent upon the manner of heating, the observations of Table I. were made. The shifting from one method of heating to another was done without permitting much alteration in the temperature of the wire.

TABLE I.

Showing Constancy of Young's Modulus under Different Conditions of Heating, Temperature Remaining the Same.

Method of Heating.	Variation in Length, Mm.	Variation in Stretch, Mm.
12.8 amperes, alternating current	.0000	.0000
12.8 amperes, direct current	.0065	.0009
External, direct current	.0059	.0009
External, alternating current	.0032	.0014
12.2 amperes, direct current	.0018	.0009

The greatest variation in length is .0065 mm. or 0.14 revolution of the micrometer head. This corresponds to a temperature variation of less than one degree Centigrade. The greatest variation in stretch is .0014 mm. This corresponds to only 0.03 revolution and is about the experimental error. These results show clearly that at this temperature, which was approximately 110° C., the stretch modulus is independent of the source of heat. These preliminary investigations and others which need not be described showed the behavior of copper to be very consistent and are the justification for the taking of more extended series of observations and the comparison of results secured at different times.

TESTS UPON SPECIMEN No. 2.

After specimen No. 2 was placed in the apparatus it was heated to a high temperature and stretched to remove the kinks. Afterward it was never heated above 150° C. The four series of readings composing Fig. 4 were taken after the wire had been heated by both methods a total of about 50 hours and after about 500 applications of the stretching weight. If such a thing were possible as a state in which all changes in elasticity, except those caused by temperature could be eliminated, this treatment should have secured the result.

As far as possible the figures have been made self-explanatory. Certain additional information which is characteristic of all the work is given in Tables II. and III. Series 4 shows a series of observations in which the heating was entirely external. In this as well as in the other three series of Fig. 4 dotted points represent observations taken while the

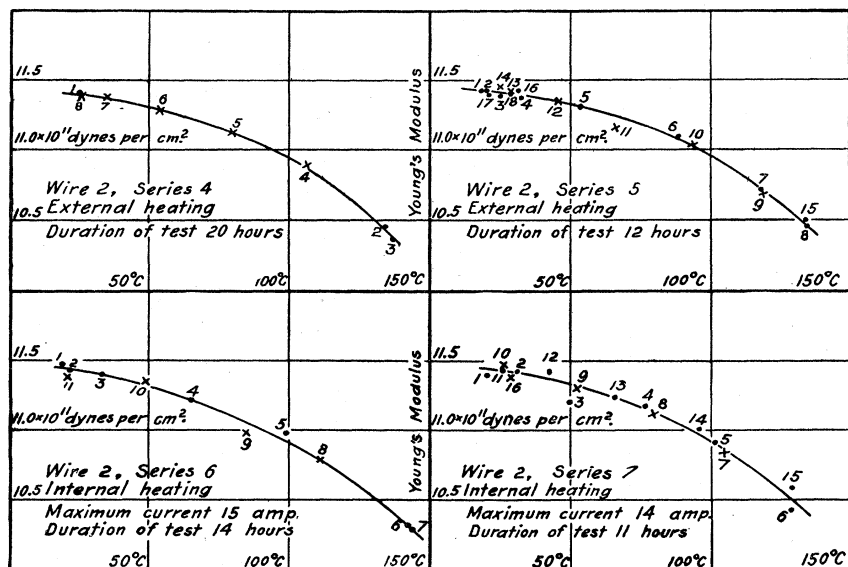


Fig. 4.

Results of tests upon specimen No. 2, showing effect of internal and external heating.

TABLE II.

Data for Series 5, Wire 2, External Heating.

Obs. No.	Time.	Temp.	Stretch (Revs. of Micr. Screw).	Young's Modulus $\times 10^{-11}$ Dynes per Cm^2 .
1	9:00 A.M.	18° C.	4.08	11.46
2	9:30	20	4.08	11.46
3	10:05	25	4.10	11.42
4	10:30	33	4.11	11.40
5	11:05	54	4.13	11.33
6	11:50	88	4.205	11.11
7	1:00 P.M.	118	4.35	10.73
8	2:00	135	4.47	10.48
9	2:45	119	4.365	10.70
10	3:15	94	4.24	11.05
11	4:10	67	4.19	11.18
12	4:40	46	4.12	11.38
13		28	4.09	11.44
14	6:00	25	4.075	11.48
15	7:15	134	4.45	10.52
16	8:00	32	4.085	11.45
17	8:45	21	4.10	11.42
18	9:30	28	4.09	11.44

TABLE III.

Data for Series 7, Wire 2, Internal Heating.

Obs. No.	Time.	Temp.	Current, Amperes.	Stretch (Revs. of Micr. Screw).	Young's Modulus $\times 10^{-11}$ Dynes per Cm ² .
1	10:30 P.M.	21° C.	0.0	4.10	11.42
2	11:15 A.M.	32	3.8	4.085	11.45
3	11:50	50	7.0	4.165	11.22
4	1:05 P.M.	77	9.8	4.18	11.20
5	1:30	102	12.0	4.28	10.93
6	2:30	128	14.1	4.49	10.43
7	3:05	105	11.9	4.31	10.85
8	3:50	80	9.5	4.20	11.13
9	4:20	53	6.8	4.13	11.33
10	5:10	27	0.0	4.07	11.50
11	6:30	27	0.0	4.08	11.46
12	7:25	43	5.0	4.09	11.44
13	8:00	66	8.0	4.15	11.26
14	8:45	97	11.2	4.245	11.02
15	9:15	129	13.9	4.42	10.60
16	9:40	28	0.0	4.10	11.42

temperature was being raised by steps and the crossed points those when the temperature was being lowered. Particular attention is called to points 2 and 3, the second of which was taken after the temperature had been maintained for twelve hours. They show that the length of time the wire was kept at the high temperature had no effect upon its elasticity. The proximity of points 8 and 1 shows the elimination of history effects and permanent changes.

Two days later Series 5 was secured, which yields results identical with those of Series 4, showing that the work of one day can be repeated at a later time. Further proof of the absence of history effects is afforded by the positions of points 14, 15, 16, 17, and 18 relative to observations taken at earlier times at the same temperatures. The fact that points 5, 6, and 7 taken with increasing temperature fall on the same curve as points 9, 10, and 12 taken with decreasing temperature shows the absence of any hysteresis effects.

Series 6 was taken three days later, followed the next day by Series 7. The procedure was the same as before except for the difference in the manner of heating. In Series 4 and 5 the wire was heated externally by means of the heating wire on the bottom of the box enclosing it. In Series 6 and 7 the wire was heated by an electric current in the wire itself. The results of Series 6 and 7 show the absence of hysteresis effects and the elimination of history effects the same as with external heating. The fact that the variation of the curves forming Fig. 4 is less than would

be expected from the experimental error shows that one is justified in taking these results as an indication of temperature effect alone and is justified in comparing the results of different days with different methods of heating.

TESTS UPON SPECIMEN NO. 3.

A third sample of the same wire was next placed in the apparatus in order to confirm the results obtained with the former specimen, to attempt still other variations in the thermal treatment and to secure the complete elastic history of a specimen. The results are given in Figs. 5, 6, and 7. Upon these curves appear every observation that was made with the third specimen, not one having been discarded.

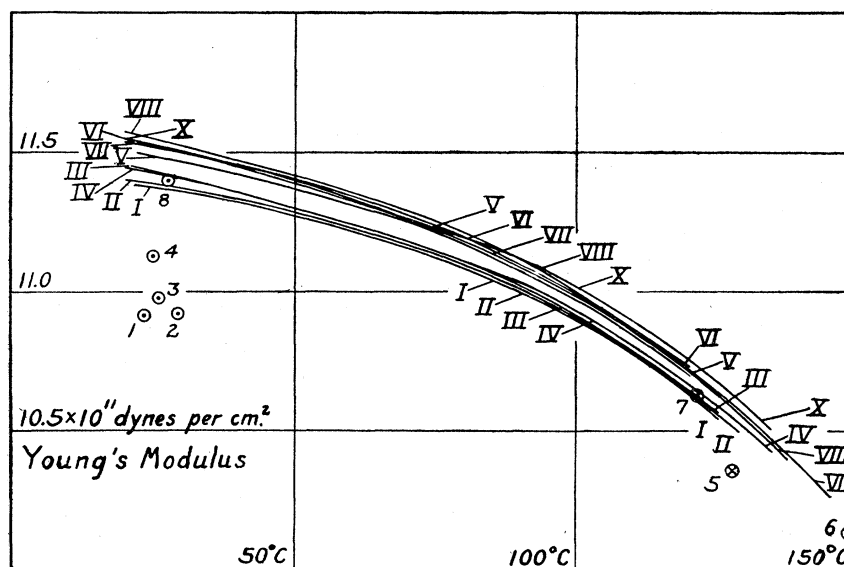


Fig. 5.

Complete elastic history of specimen No. 3. The curves are those of Figs. 6 and 7, the Roman numerals corresponding to the series numbers.

First Changes in Elasticity.—The specimen was made as free from kinks as possible and placed in the apparatus. Observation 1, Fig. 5, was at room temperature. The wire was then heated externally to about 125° C. and cooled. Observation 2 shows no change. Heating by 17 amperes current to about the same temperature brought a slight increase in the modulus as indicated by observation 3. There was very little increase in length during this treatment, as the additional weight was not applied at high temperatures.

It was anticipated that continued application of the additional weight while the wire was hot would remove any kinks in the wire, cause an increase in length and a change in the modulus. The wire was heated by 17 amperes current and the weight applied a number of times. On

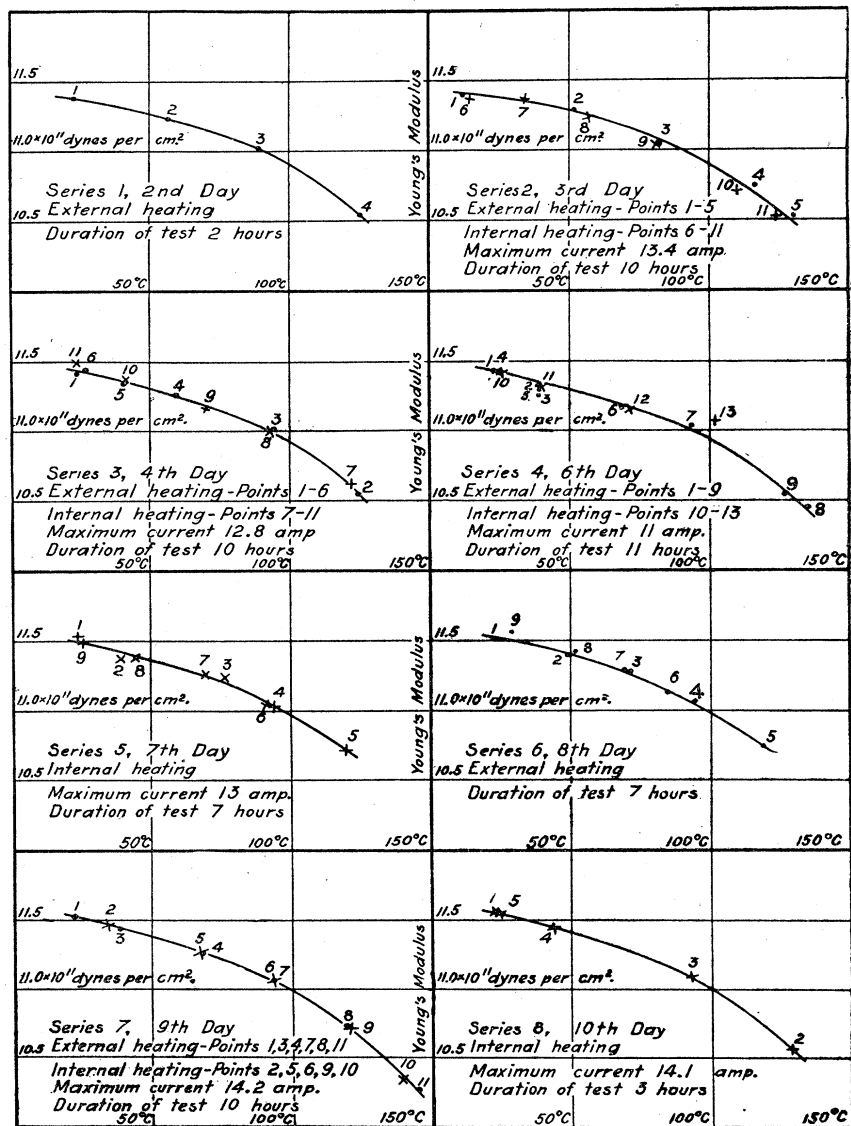


Fig. 6.

Results of tests upon specimen No. 3 with different thermal treatment, showing history effect, change of Young's modulus with temperature, and absence of any effect peculiar to current heating.

cooling an increase of length of about 0.2 mm. was found and an increase of Young's modulus as indicated by observation 4. The wire was again heated to 125° C. and Young's modulus measured at this temperature, resulting in observation 5. The temperature was then increased to about 150° C. and the additional weight applied about thirty times, resulting in observation 6. Cooling to 122° C. gave observation 7, and to 28° C., observation 8. It was believed that a sufficiently steady state had been secured to justify the trial of a series of readings and a comparison of points 6, 7, and 8 with reference to later observations shows this to have been the case. It is highly probable that the increase in Young's modulus due to this particular treatment was caused as much by the mechanical stretching and straightening of the wire as by the thermal treatment, although the latter was undoubtedly essential to the securing of the steady state.

Discussion of Fig. 6.—Following the work just described came eight days of testing under a variety of conditions. Fig. 6 shows the results and has been made practically self explanatory. The order in which the observations at the different temperatures were taken and the manner of heating were varied in a number of ways in order to catch any possible changes caused by treatment. Series 1 is with external heating, increasing temperature. Series 2, first part, was taken in exactly the same manner and the results show that for external heating a sufficiently steady state had been reached so that only small permanent changes were to be anticipated. The second part of Series 2 was taken in exactly the same manner except for the change in the manner of heating, the external heating having been changed to internal by means of the current. No difference is observed. The following day a series was taken in a similar way except that the observations were made with decreasing temperature. The nature of the temperature change remains the same and a slight permanent increase appears.

In Series 4 the systematic heating and cooling over the whole temperature range was not followed, but no peculiar effects were observed. During the following night the wire lay unstretched and it is possible that this had some influence in causing the permanent increase of elasticity observed the next day. Series 5 is the result of heating and cooling with internal heating and Series 6 is for the same treatment with external heating. No hysteresis effect appears nor does there appear any difference due to method of heating.

Discussion of Series 7.—Series 7 is the most interesting single set of observations, and was taken in order to compare the two methods of heating without permitting the elapse of several hours or allowing an

appreciable change of temperature. At each temperature at which observations were taken, the change in heating was made gradually, without permitting the temperature of the wire to change. Observation 1 was at room temperature. Observation 2 was with current heating. Without permitting more than a few degrees variation in temperature the current in the wire was decreased while that in the heater was increased. Observation 3 was with external heating. The temperature was then increased and observation 4 taken. Again the method of heating was gradually changed until observation 5 was secured with internal heating at practically the same temperature. In the same way 6 and 7; 8 and 9; 10 and 11 were taken. The results show in a striking manner that the changes accompanying current heating are only temperature effects. The tests of the tenth day with internal heating gave Series 8.

In any single series of this group the permanent changes with treatment do not appear prominent. But a comparison of the eight curves, Fig. 5, reveals a gradual increase of Young's modulus from day to day. This was caused undoubtedly by the continued application of the additional weight which caused a very small gradual stretching.

Effect of the Period of Rest.—Two days after the observations of Series 8 a measurement was taken at room temperature after which the wire was undisturbed for 106 days. This last observation appears with Series 9, Fig. 7, and a comparison with point 1 shows a slight recovery of

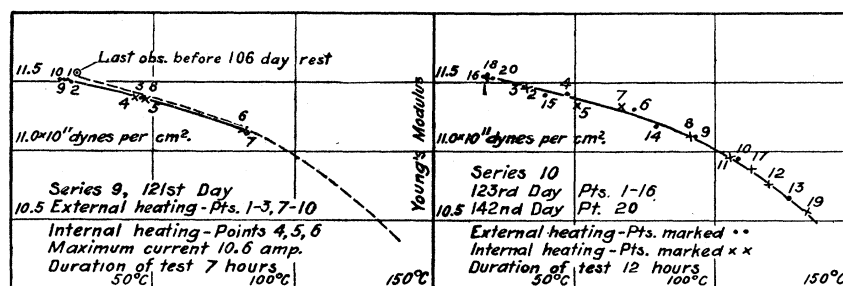


Fig. 7.

Results of tests upon specimen No. 3 after 106-day rest.

the wire during the long rest. Nine other observations were then taken with an idea of determining whether or not the modulus would remain low upon heating. This proved to be the case both for external and current heating as is clearly brought out by comparison with the dotted line which is the Series 8 curve. Two days later, as shown by Series 10, further heating and testing brought the specimen back to its former state

for low temperatures although at high temperatures greater values of Young's modulus were observed than ever before. Too much stress must not be laid upon these slight changes in the wire. The difference between Series 9 and Series 8 is small and may have its origin in a slight change in the working condition of the apparatus caused by its period of idleness. The important fact is that no great change occurred and the wire was in practically the same condition as when left. The explanation of the difference in slope in some of the curves is that the experimental error is of about the amount of the variation. The total change in Young's modulus over the entire temperature range is only 10 per cent. and the difference in stretch which this represents corresponds to less than one revolution of the micrometer head. In order to show this variation the scale to which Young's modulus is plotted has to be so large that the errors of observation become appreciable. When the magnitude of the probable error is taken into consideration the consistency of the curves taken under such a variety of conditions is remarkable.

CONCLUSIONS.

The following conclusions apply to the sample of copper wire that has been studied. I am of the opinion that other specimens of copper will yield results differing only in the magnitude of the changes, and believe that the results are very suggestive of the effects that can be expected in other metals. But work with iron wire now under way and certain results of other investigations to which reference is made later makes me feel that too great caution cannot be exercised in making general conclusions from observations upon particular samples.

From the preliminary observations and the tests upon specimens Nos. 2 and 3 we reach these conclusions:

1. A copper wire can be brought to a steady elastic state in which the stretch modulus becomes a function of temperature. In the present instance it was brought about by repeated stretching and heating and cooling over a range of temperature of 20° C. to 150° C. The preliminary tests show that the rate at which the heat was applied or at which the wire was cooled, the length of time the temperature was maintained, and the thermal route by which a given temperature was reached had no apparent effect upon the value of the modulus at any given temperature. These same facts are brought out still more convincingly by the extended work upon specimens 2 and 3, which also show that permanent changes in the modulus can be practically eliminated by a relatively small amount of heating and stretching, which if continued for a longer time renders them negligible.

2. Heating by an electric current has no effect other than that caused by the accompanying temperature. This was proved for one temperature by the results of Table I. and at a number of different temperatures by the work of Series 7, Wire 2, shown in Fig. 6, in which the heating was alternately by current and the external source.

Further proof with step by step heating is also furnished by the identical nature of the results recorded in the four curves of Fig. 4, two curves of which are for current and two for external heating. Conclusive evidence is afforded by the long series of experiments carried out upon the third specimen, in which every possible way of varying the methods of heating was attempted without detecting any difference in the results.

3. Young's modulus of a copper wire decreases with increase of temperature at an increasing rate. In the case of the specimen studied, the amount of decrease at different temperatures in terms of per cent. of the value of the modulus at 20° C. is as follows: 40° C., 0.6 per cent.; 60° C., 1.7 per cent.; 80° C., 2.9 per cent.; 100° C., 4.6 per cent.; 120° C., 7.0 per cent.; 140° C., 10.0 per cent. At 20° C. the modulus of the sample in the steady elastic state was 11.5×10^{11} dynes per cm².

The true nature of the temperature change was first shown by the work with specimen 2 as represented by the curves of Figs. 4 and was fully substantiated by the further work with specimen 3 which yielded identical results.

4. Heating and repeated stretching increases Young's modulus. As is shown by the results with specimen 3 there is an apparent increase of about 5 per cent. caused by the first four heatings and the accompanying stretching, which is thought to be due in part to the straightening out of the kinks. But there was a further increase of 2 per cent., which was a true history effect, although caused probably as much by the repeated stretching as by the heating.

COMPARISON WITH RESULTS OF OTHERS.

As has already been stated previous results upon the effect of current do not agree. A comparison of these with the present work will not be attempted as the degree of accuracy in the latter is far greater. The conclusion that there is no effect of current upon copper other than that caused by temperature can be accepted to an accuracy of less than 1 per cent.

An interesting comparison regarding the history effect and the magnitude of change with temperature is furnished by the work of Shakespeare,¹

¹ Loc. cit.

who used interference methods to determine the modulus at 13° C. and at 100° C. He does not compute the value of the modulus but gives the change from 13° C. to 100° C. as 3.6 per cent. For this range the present specimen shows a change of 5 per cent. Shakespeare also found that the modulus increased with continued heating and cooling, until a permanent state was reached. The first heating produced a sudden increase of several per cent. which was followed by a gradual change. Nine heatings produced a total increase of 12 per cent. The permanent change described in this paper was of the same kind and amounts to 7 per cent.

In the following comparison of results upon the temperature effect the bracketed figures are taken from our own curves. At 20° C. our value of Young's modulus was 11.5×10^{11} dynes per cm². Miss Noyes¹ found the modulus of a specimen of copper to be 12.02 at 20° C. This and another specimen tested at temperatures up to 150° C. gave a uniform decrease of the modulus which computed for 20° C. to 140° C. gives a change of 8.5 per cent. and 16 per cent. respectively [10 per cent.]. At 17.4° C. Gray, Blyth, and Dunlop² found the modulus of a sample of commercial copper to be 11.15 and of hard drawn electrolytic copper at 19.5° C. to be 12.9. They made tests over a range of 100° C. and reported a uniform rate of decrease which computed for 20° C. to 100° C. gives for the two samples changes of 1.2 per cent. and 3.4 per cent. respectively [4.6 per cent.].

Slotte³ found for two specimens changes of 6.6 per cent. and 4.2 per cent. respectively [1.5 per cent.], for a temperature increase from 20° C. to 60° C. The moduli at 20° C. were respectively 12.4 and 12.7. The more important point is that he did not find a uniform rate of change. The modulus of the former specimen decreased at an increasing rate while the latter shows a decrease at a decreasing rate. Decrease of the modulus at an increasing rate is also indicated by the work of Wertheim.⁴ If his results at 15° C., 100° C., and at 200° C. are plotted and a smooth curve drawn it is almost identical with our own curves, except that the values are about 10 per cent. lower. Regarding the nature of the change observations on the torsion modulus are valuable although too great stress must not be placed on them. Kohlrausch and Loomis⁵ made observations upon copper at various temperatures. They found an increasing rate of change as the temperature increased. Computed for 20° C. to 100° C. the change is 4.2 per cent. [4.7 per cent.], and for 20° C. to 140° C. 7.5 per cent. [10 per cent.]. Slotte⁶ has also made a study

¹ Loc. cit.² Loc. cit.³ Loc. cit.⁴ Loc. cit.⁵ Loc. cit.⁶ Loc. cit.

of the torsion modulus of copper and finds the same increasing rate of change. Corresponding values from his work are 5.9 per cent. and 8.3 per cent.

The results that have been cited must be considered from two standpoints, first as to the general nature of the changes of Young's modulus, whether uniform or otherwise, and secondly, regarding the numerical values.

Examination of the methods of those who have reported a uniform decrease shows that their work was not of sufficient accuracy to have detected the variation from a linear relation found by others. We are not certain how much importance should be attached to the work of Slotte upon Young's modulus. He worked also with aluminum, iron, and platinum as well as copper and found a decreasing rate of change. None of these results seem to have been repeated. The torsion modulus can be determined to a much higher degree of accuracy and Slotte's recent work upon the torsion modulus of all these metals shows an increasing rate of change. Since the evidence that is available points toward a similarity in the nature of the temperature change of the two moduli this raises a question as to whether Slotte's former results upon Young's modulus are a true temperature effect.

With the results of Wertheim and of Slotte upon one specimen agreeing with our own and the work of Kohlrausch and Loomis and of Slotte upon the torsion modulus showing the same general type of change, I believe that the conclusion that Young's modulus of copper decreases with increase of temperature at an increasing rate has sufficient corroboration.

A comparison of the numerical values of Young's modulus and of the temperature coefficient, ample allowance being made for experimental error, leaves no doubt as to the importance of chemical purity, method of preparation, and size of specimen. It is possible that samples of copper may be found that will show effects peculiar to current heating, or will not give modulus-temperature curves of the simple form that I have found, but I am inclined to believe the main differences will be found in the magnitude of the coefficients. Undoubtedly further work will reveal some relation between the chemical composition, the coefficient of linear expansion, the electrical conductivity, and other physical constants, and the value of Young's modulus and its rate of change with temperature. Investigation is under way upon other metals in order to study the effect of current, to determine the temperature coefficient of Young's modulus, and to study the relation between the temperature changes of the two elastic moduli.

In conclusion I wish to acknowledge indebtedness to the staff of the physical laboratory of the State University of Iowa for their interest in the work and especially to Professor G. W. Stewart for suggesting the problem and to Professor J. N. Pearce, of the department of chemistry, for the chemical analysis.

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