Second Series *February, 1922* *Vol. XIX, No. 2*

THE

PHYSICAL REVIEW.

ON THE GAMMA-RAY ACTION OF EXTENSIVE FLAT RADIUM PREPARATIONS AT DIFFERENT DISTANCES WITH AND WITHOUT ABSORBING MATERIALS.

By Victor F. Hess.

Synopsis.

Intensity of gamma-radiation from a radium-covered disk at points along the central normal, with and without absorbing layers, is of interest in connection with the use of radium applicators for gamma-ray treatments. By integrating the effects of narrow rings of the active material, a *theoretical formula is derived* for the intensity at a distance p from a disk of radius R if the rays go through one or more filters whose absorptions are known and through a thickness of tissue h_0. This formula involves the so-called exponential integral for which tables are available. Numerical values have been calculated for certain special cases and these are given in two tables which enable the intensity to be determined at various distances from any uncovered disk in air and also from any disk covered by a layer of brass 2 mm. thick. A given amount of radium spread on a 4 cm. disk gives at distances greater than 4 cm. about the same intensity as when concentrated on the end of a small tube, but the effect near the active layer is over ten times less intense in the former case. This result shows that burns of the skin may be avoided by using sufficiently large disks without decreasing the depth action materially.

New unit of gamma-ray intensity is proposed, to be called a *milli-eve* and to be defined as the intensity at a distance of 1 cm. from a point-shaped radium preparation of one milligram.

Introduction.

THE "flat radium applicators" consist mostly of a metallic disk covered on one side with a lacquer, with which some radium salt is mixed. The shape of the disk may be rectangular or circular. The calculation is given for circular disks and may be applied for other shapes with sufficient accuracy. In order to cut off all alpha and most of the beta rays the surface of the radium disk is covered by a metallic disk. This "filter-disk" consists of brass, copper or similar materials and has a thickness of 1–2 mm. A second filter, made of a material of low atomic weight (celluloid, rubber, etc.) is applied to absorb the secondary β-rays, which come out from the metal disk. The thickness of this second disk is relatively small (1–3 mm.). The absorption of the primary γ-rays in this layer may therefore be neglected.

Theory of the General Case.

Fig. 1 illustrates the general case. The radioactive layer (circular area of radius R) is considered as a disk of infinitely small thickness and σ represents the quantity of radium per sq. cm. The metallic filter has a thickness d_0 and is applied directly below the radioactive disk.

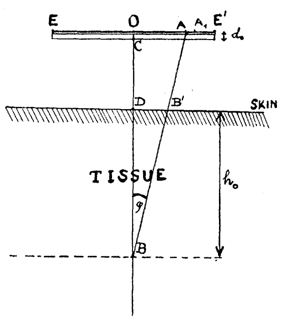

Fig. 1.

The radius of the disk is

$$R = OE, \qquad R = OE'.$$

The point at which the intensity of the γ-rays is considered is B.

We call the distances

$$OA = r, \qquad OB = p,$$
$$OC = d_0, \qquad BD = h_0.$$

The last-mentioned letter indicates the thickness of tissue, which has to be traversed before point B is reached.

Now we take an arbitrary point A on the radioactive disk and consider the action of the ring-shaped element of area of the width $AA_1 = dr$ at the point B.

The action of the γ-rays of this element at B without any absorption would be

$$dQ = K \cdot \frac{2\pi r \cdot dr \cdot \sigma}{p^2 + r^2},$$

where K indicates the so-called "Ève's number"; that is the number of pairs of ions, generated by the γ-rays of 1 g. of radium at the distance 1 cm. from the source of rays.

The γ-rays coming from A traverse the metallic filter and the tissue in an oblique direction according to an angle $\varphi = \measuredangle OAB$. We call the absorption coefficient in the metal-filter μ, in the tissue ν.

The factors which express the real absorption of γ-rays in the filter and the tissue become therefore

$$e^{-\frac{\mu d_0}{p}\sqrt{{}^2p+r^2}} \quad \text{and} \quad e^{-\underline{\nu h_0}\sqrt{p^2+r^2}}.$$

Their product corresponds to the total absorption.

Thus we get for the real intensity of γ-rays at point B, created by the rays of the ring-shaped element dr, the expression

$$dQ = 2\pi K\sigma \frac{rdr}{p^2 + r^2} \cdot e^{-\frac{\mu d_0 + \nu h_0}{p}\sqrt{p^2+r^2}}.$$

For simplicity we may write

$$\frac{\mu d_o + rh_o}{p} = b.$$

This expression represents a constant for every special distance.

To find the total amount of the γ-ray intensity at B we have to integrate dQ over the whole area of the disk. Thus we obtain the integral

$$Q = 2\pi K\sigma \int_0^R \frac{re^{-b\sqrt{p^2+r^2}}}{p^2 + r^2} dr. \qquad (1)$$

It is easy to reduce this integral to a known form by introducing the new variable $u = b \cdot \sqrt{p^2 + r^2}$.

Therefore the integral becomes

$$Q = 2\pi\sigma K \int_{b.p}^{b.\sqrt{p^2+r^2}} \frac{e^{-u}}{u} du \qquad (2)$$

The integral $\int (e^{-u}/u)du$ is commonly called "the exponential integral." Numerical values of this integral are given by J. W. L. Glaisher,[1] W. L. Miller and T. R. Rosebrugh [2] and others.

We call as usual

$$-\int_a^\infty \frac{e^{-u}}{u} du = Ei(-a).$$

The corresponding value of this integral may be taken directly from the tables for all values of a. A transformation of formula (2) leads to the expression

$$Q = -2\pi\sigma K[Ei(-b\cdot p) - Ei(-b\cdot p\sqrt{p^2 + R^2})].$$

After substituting for b we obtain the general solution

$$Q = 2\pi\sigma K\left[Ei\left(-\frac{\mu d_0 + \nu h_0}{p}\sqrt{p^2 + R^2}\right) - Ei(-\mu d_0 - \nu h_0)\right]. \qquad (3)$$

By introduction of the numerical values we obtain thus directly the ionization created by the γ-rays at the chosen distance from the disk after allowing for the absorption in the filter and the tissue. When the values of Q for different distances p are calculated the law of decrease of intensity of the gamma-radiation in each special case may be obtained.

[1] J. W. Glaisher, Phil. Trans., Vol. 160, p. 367 (1870).

[2] W. L. Miller and T. R. Rosebrugh, Trans. Roy. Soc. of Canada, 3 Series, Vol. 9, sect. 3, p. 73 (1903).

Formula (3) may be easily applied also in cases, where several different filters are used. In these cases we have simply instead of μd_0 to introduce the sum $(\mu d_0 + \mu' d_0' + \cdots)$ where μ and d_0 denote the absorption coefficients and respective thicknesses of the different filters used.

It can also be seen from formula (3) that Q is independent of the unit of length used for p and R; that it depends only on the ratio $\frac{p}{\sqrt{p^2+r^2}}$, which is the cosine of the angle subtended by the radius of the disk. Therefore the effect at 4 cm. from a disk of 2 cm. diameter is the same as at a distance of 2 cm. from a disk of 1 cm. diameter. This holds of course only when σ, the quantity of radium per unit of area, is kept constant. In the numerical tables which are given later not σ but the total radium content of disks of different diameters is taken as constant. This corresponds more to the actual conditions of medical use.

Different absorbent materials have the same effect, when the products of absorption coefficient and thickness of layer have the same numerical value. Thus a layer of brass (thickness d_0) and a layer of tissue (h_0) are equivalent, when $\mu \cdot d_0 = \nu \cdot h_0$.

Special Cases.

1. Action of the γ-rays of a radium disk, covered by a metal screen (thickness d_0) in air.

This case is of interest, for it gives the actual amounts of ionization. by the γ-rays of the disk, when no tissue is between the source and the chosen point (for instance a point directly on the surface of the skin). This case corresponds to $h_0 = 0$ and formula (3) changes into the simpler form

$$Q = 2\pi\sigma K\left[Ei\left(-\frac{\mu d_0}{p}\sqrt{p^2+r^2}\right) - Ei(-\mu d_0)\right]. \qquad (3')$$

2. Action of the radium disk without any absorbing filters: This case is physically not without interest: from this it is possible to judge upon the difference in the distance-action of a disk and an approximately point-shaped source of radiation. For $d_0 = h_0 = 0$ formula (3) becomes indefinite. Thus we have to go back to formula (1). Here the exponential factor becomes 1 and the integral leads to the simple solution

$$Q = 2\pi\sigma K\left[1\cdot 1513 \cdot \log_{10} \cdot \frac{p^2+R^2}{p^2}\right] \qquad (1')$$

(for greater convenience in practical cases Briggs's logarithms are introduced).

Practical Applications and Proposal of a New Unit for the Intensities of Action of the γ-Rays.

In the numerical computations of the formulas (1)–(3) given above it will be found troublesome to express Q in terms of K (number of pairs of ions, created per c.c. and sec.), because K is a very high number (*Eve* found, that in air $K = 3 \cdot 9.10^9$ ions/ccm. sec.). In most cases relative numbers, for instance the expressions within the brackets, would suffice.

However, for a greater convenience of use, expecially for physicians, a new unit for γ-ray intensities may be proposed: The action (ionization) of the γ-rays of RaC in equilibrium with 1 gram of radium, given as a nearly point shaped source (very small tube) at the distance of 1 cm. from this source may be called "*1 Eve.*" The one thousandth part of this intensity be called "*1 Milli-Eve.*" According to this definition[1] a small radium-tube of 50 mg. radium element would give (by its γ-radiation) in the distance 1 cm. 50 Milli-Eves; in 2 cm. (corresponding to the inverse square of distance-law, 12.5 Milli-Eves; in 3 cm. 5.56 Milli-Eves; in 0.5 cm. 200 Milli-Eves, etc.

According to that the total amount of γ-radiation used in one exposure may be expressed in Milli-Eve hours, that is the product of the number of Milli-Eves into the number of hours of exposure.

To express the results of the formulas of this paper in Milli-Eves, we have to consider that according to the definition of this new unit the intensity of a point-shaped and unfiltered radium preparation at 1 cm. distance has to be taken as 100 when a source of 100 mg. is used. When we use a filtration of 2 mm. brass, the intensity at the same distance would be about 90 and all other intensities have to be changed in the same proportion.

In the following tables it is assumed that 100 mg. preparations are used. For other amounts the figures have to be altered accordingly.

The values of the exponential integral $Ei(-a)$ are taken from abbreviated tables, given in Meyer and Schweidler's "Radioactivity" (Teubner, Leipzig, 1916). The coefficient of absorption of the complex γ-radiation of RaB and RaC has been assumed as $\nu = 0.094$ cm. This is an average of experimental values found by G. Failla (Amer. Journ. of Roentgen., Vol. 3, 215–232, 1920) and A. Fernau (Strahlentherapie, Vol. 9, p. 236–254, 1918).

Table I. shows the γ-ray-intensities at different distances from a circular disk (4 cm. diameter), covered uniformly with 100 mg. radium for three cases: without any filter (formula 1′), with a filter of 2 mm.

[1] Prof. A. S. Eve (Montreal, Canada) was the first who determined the absolute values of the ionization of γ-rays at different distances from a known radium source. A. S. Eve. Phil. Mag. (6), 12, 189 (1906); 22, 551 (1911); 27, 39 (1917).

brass (formula 3′) in air and in tissue. Column 3 gives the figures in the case, when the same filtered disk is directly applied above the skin. All figures in this column are considerably lower than in column 2 according to the absorption in the tissue.

TABLE I.

Radium Disk, 4 cm. diameter, 100 mg. Radium.

Distance from the Surface of the Radium Disk (Cm.).	γ-ray Intensities in Milli-Eves.		
	No. 1, Uncovered Disk in Air.	No. 2, Disk Filtered by 2 Mm. Brass in Air.	No. 3, Disk Filtered by 2 Mm. Brass in Tissue.
$p =$ 0.3	107.4	81.2	79.7
0.5	80.0	64.9	61.5
1.0	45.4	38.9	35.4
2.0	19.5	18.1	14.9
3.0	10.3	8.5	6.8
4.0	6.3	5.1	4.2
5.0	4.2	3.4	2.7
6.0	3.0	2.3	1.6
7.0	2.1	1.8	1.1
8.0	1.7	1.4	0.8
10.0	1.1	0.8	0.5

As a second example the intensities of the γ-radiation have been calculated for circular disks of the same radium amount but different diameters, varying from 0.4 to 8.0 cm. The smallest type of these disks corresponds to a practically point-shaped radium-source (very small tube). The figures in the following Table II. are again given in Milli-Eves.

TABLE II.

Distance from the Surface of the Radium Disk in Cm.	γ-ray Intensities (in Milli-Eves) (all Disks Filtered by 2 Mm. Brass).				
	Disk of 0.4 Cm. Diameter.	Disk of 1.0 Cm. Diameter.	Disk of 2.0 Cm. Diameter.	Disk of 4.0 Cm. Diameter.	Disk of 8.0 Cm. Diameter.
$p =$ 0.3	888	528	215	81	24
0.5	341	289	154	65	21
1.0	90	82	72	39	16
2.0	22	22	21	18	9.7
3.0	10	10	9.5	8.5	6.4
4.0	5.6	5.7	5.5	5.1	4.5
5.0	3.6	3.6	3.6	3.4	3.1
6.0	2.5	2.4	2.4	2.3	2.2
7.0	1.8	1.9	1.8	1.8	1.7
8.0	1.4	1.4	1.4	1.4	1.3
10.0	0.8	0.8	0.8	0.8	0.8

It is easily to be seen from this table that the intensities of the γ-rays of the smallest and the largest disk on the surface vary in the proportion 888 : 24; the action in the depth (from 3 cm. on) is nearly the same with all the disks. The use of large radium disks is therefore recommendable in all cases, where the action of points near the preparation itself should be as small as possible (for instance to avoid burns of the skin).

Physical Research Laboratory,
The United States Radium Corporation,
Orange, N. J.,
October, 1921.