THE WIDTH OF X-RAY SPECTRUM LINES.

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SYNOPSIS.

Width of X-ray Spectrum Lines.—(1) *Four causes* are discussed: (*a*) the width of the slit employed, (*b*) angular imperfections of the crystal, (*c*) finite resolving power of the crystal grating, and (*d*) lack of homogeneity of the x-rays. The effect of the first two causes is independent of the angle of reflection and therefore the same for all orders and all lines; the effect of the third is shown to vary as $1/\sin\theta\cos\theta$ and therefore to decrease as θ increases up to 45°; while the effect of the last is equal to $(d\lambda/\lambda)\tan\theta$ and therefore increases with θ. (2) *Measurements of two tungsten lines,* λ 1.242 and 1.279 Å, in the first four orders from the cleavage faces of calcite and rock-salt, are given. The fact that in both cases the width is least for the second order and greatest for the fourth order shows that both the third and fourth causes contribute measurably to the observed width, as well as the first two.

Lack of homogeneity of tungsten x-ray lines computed from the above measurements comes out greater than $0.0007\,\lambda \pm .00013\,\lambda$, which is about 0.5′ for the first order. This cannot be explained as a Doppler effect or as due to the damping of the electronic motion; however it is in accord with the complexity of the lines as predicted by Sommerfeld's theory of elliptic electronic orbits.

SEVERAL years ago the writer performed a series of experiments designed primarily to measure the relative intensities of the different orders of x-ray spectrum lines reflected from crystals of rock-salt and calcite. A recording x-ray spectrometer, which has previously been described,[1] was employed, the intensities of the lines being taken proportional to the area under the curves on the record representing the different spectrum lines. These intensity measurements, in which cleavage faces of the crystals were employed, were later found to be valueless as a test of the theory of reflection, because of the selective absorption which occurs at the angle of maximum reflection from a cleavage face.[2] The results show, however, an interesting broadening of the lines in the higher orders.

The width of the spectrum lines was measured at a point midway between the peak of the curve and a base-line representing the intensity of the general radiation. The average values thus found for the tungsten lines $\lambda = 1.242$ and $\lambda = 1.279$ Å.U. are as follows:

[1] A. H. Compton, PHYS. REV., 7, 658 (1916).

[2] A. H. Compton, PHYS. REV., 10, 95 (1917); W. L. Bragg, James and Bonsanquet, Phil. Mag., 41, 309 (1921).

TABLE I.

Width in Minutes of Arc at Mid-point of Line.

Crystal.	First Order.	Second Order.	Third Order.	Fourth Order.
Calcite	16.3 ± .6	16.0 ± .36	17.3 ± .7	18.3 ± .12
(Calc.)	15.4	(16.0)	16.7	(18.3)
Rock-salt	20.1 ± .5	18.9 ± .3	20.3 ± .6	21.2 ± 1.2
(Calc.)	18.3	(18.9)	19.8	21.5

The size of the slits remained unchanged throughout the experiments. The measurements on the second and fourth orders of reflection from calcite are the most valuable, because these two lines are of nearly equal intensity, which tends to eliminate any consistent errors.

The observed width of a spectrum line may result from four different causes, (*a*) the angular aperture of the slits, (*b*) angular faults in the crystal grating, (*c*) the finite resolving power of the grating, and (*d*) the lack of homogeneity of the incident x-rays.[1] The width due to the size of the slits will obviously be the same at all angles. That due to angular faults in the crystal may vary somewhat according to the part of the crystal which is exposed to the x-rays, but should on the average be the same for all angles of reflection. It is clear that no crystal will act as a perfect grating, and that the resolving power will depend upon the size of the portions of the crystal which are effectively perfect. It can be shown[2] that if these perfect portions are assumed to be parallelopipeds of width along the crystal face δx, height δy and depth into the crystal δz, the effective width of the line (area/height) in radians is:

$$(1) \qquad \delta\theta_c = \frac{\lambda}{2\delta x \sin\theta} \quad \text{for} \quad \frac{\delta x}{\delta z}\tan\theta < 1,$$

$$\delta\theta_c = \frac{\lambda}{2\delta z \cos\theta} \quad \text{for} \quad \frac{\delta x}{\delta z}\tan\theta > 1,$$

where λ is the wave-length and θ the glancing angle. In crystals of rock-salt and calcite δx and δz are doubtless on the average about equal, so that $\delta\theta_c$ will be a minimum for $\theta = \pi/4$, increasing symmetrically for smaller and larger angles. Because of chance variations in δx and δz

[1] In the experiments here described, the beam incident upon the crystal was collimated by a pair of narrow slits, and the opening at the ionization chamber was wide enough to admit all the reflected rays. With Bragg's method of using narrow slits near the x-ray tube and at the ionization chamber and a wide slit at the crystal, the width also depends upon a fifth factor, the penetration of the x-rays into the crystal.

[2] This follows (after considerable reduction) from the results given by the writer in THE PHYSICAL REVIEW, 9, 37 (1917).

for the different component crystals, the change from $\delta\theta_c \propto 1/\sin\theta$ to $\delta\theta_c \propto 1/\cos\theta$ will be gradual. Thus it appears that the width of the line due to this finite resolving power should vary approximately according to the expression

$$\delta\theta_c \propto 1/\sin\theta\cos\theta. \tag{1a}$$

The angular width due to a given range of wave-lengths $\delta\lambda$ in the incident x-rays may be found by differentiating $\lambda = 2D\sin\theta/n$ with respect to θ. Thus we find that

$$\delta\theta_d = \frac{n\delta\lambda}{2D\cos\theta} = \frac{\delta\lambda}{\lambda}\tan\theta. \tag{2}$$

The total width of the line is a rather complicated function of $\delta\theta_a$ due to the width of the slits, $\delta\theta_b$ due to angular imperfections of the crystals, and $\delta\theta_c$ and $\delta\theta_d$ given by equations (1) and (2) respectively. Thus we may write

$$\delta\theta = F(\delta\theta_a, \delta\theta_b, \delta\theta_c, \delta\theta_d).$$

The function F will increase with every increase in any variable $\delta\theta_r$, in such a manner that $\delta\theta$ will be greater than any one of its components but will be less than the sum of all four. The form of this function cannot be determined unless we know the exact manner in which the intensity varies with the angular distance from the center of the line on account of each factor contributing to the width. Lacking such definite knowledge, a simple graphical analysis shows that an increase in $\delta\theta_r$ will result in an increase in $\delta\theta$ of the same order of magnitude but smaller than the change in $\delta\theta_r$, *i.e.*,

$$1 > \frac{\partial\delta\theta}{\partial\delta\theta_r} > 0.$$

Furthermore by Taylor's theorem, to a first approximation,

$$\delta\theta = \delta\theta_0 + \frac{\partial\delta\theta}{\partial\delta\theta_a}\Delta\delta\theta_a + \frac{\partial\delta\theta}{\partial\delta\theta_b}\Delta\delta\theta_b + \frac{\partial\delta\theta}{\partial\delta\theta_c}\Delta\delta\theta_c + \frac{\partial\delta\theta}{\partial\delta\theta_d}\Delta\delta\theta_d, \tag{3}$$

where $\Delta\delta\theta_r$ represents a change in $\delta\theta_r$.

In the experiments using a calcite crystal, $\delta\theta_a$ and $\delta\theta_b$ remain constant, $\delta\theta_c$ decreases with θ (up to 45°), and $\delta\theta_d$ increases with θ. The observed increase in the width of the lines in the higher orders therefore means that $(\partial\delta\theta/\partial\delta\theta_d)\Delta\delta\theta_d$ is more prominent than $(\partial\delta\theta/\partial\delta\theta_c)\Delta\delta\theta_c$. If we neglect the term in $\delta\theta_c$ and compare the value of $\delta\theta$ at two angles θ_1 and θ_2, it follows from expression (3) that

$$\delta\theta_2 - \delta\theta_1 = \frac{\partial\delta\theta}{\partial\delta\theta_d}(\delta\theta_{d_2} - \delta\theta_{d_1})$$

whence according to equation (2)

$$\frac{\delta\lambda}{\lambda} = \frac{\partial\delta\theta_d}{\partial\delta\theta} \frac{\delta\theta_2 - \delta\theta_1}{\tan\theta_2 - \tan\theta_1}. \tag{4}$$

Substituting the values of $\delta\theta$ observed in the second and fourth orders from calcite, noting that the angles θ_2 and θ_4 are 24.2° and 55.2° respectively and that $\partial\delta\theta_d/\partial\delta\theta$ is of the order of but greater than unity, we find that $\delta\lambda/\lambda$ is of the order of but greater than $0.0007 \pm .00014$. The values calculated in the second row of Table I. result from the further approximation that $\partial\delta\theta_d/\partial\delta\theta$ is constant for all values of $\delta\theta_d$. If we take this constant as unity, we find 0.0007 for the minimum value of $\delta\lambda/\lambda$, and 0.5′ for the minimum value of the width of the first order spectrum line due solely to lack of homogeneity of the x-rays. The observations on rock salt lead to values of the same order of magnitude.

The widths calculated for the lines reflected from calcite agree well with the experiments except in the first order. This line appears to be too broad, as we have noticed should result from the small resolving power if the crystal is sufficiently imperfect. The imperfection of the rock-salt crystal as compared with calcite shows itself both in the greater value of the constant $\delta\theta_b$ for rock salt, indicated by a greater width in all orders, and by the greater difference between the breadth of the first and second orders. According to equation (1*a*) the broadening due to this imperfection is nearly proportional to $1/\sin\theta\cos\theta$, *i.e.*, to 2.5, 1.3, 1.0 and 1.1 in orders 1, 2, 3 and 4 respectively. A difference in breadth of 1.8′ between the first and second orders of rock salt[1] will therefore correspond to a difference of only 0.3′ between the second and fourth orders. The unmistakable broadening of the higher orders can apparently be accounted for only by a true lack of homogeneity of the x-ray spectrum line. Thus it appears that all the terms of expression (3) have values that are appreciable in experiment.

It is of interest to consider the possible origin of the observed non-homogeneity of the x-rays. Let us consider first the width to be expected on the basis of the Doppler effect. The average velocity of the tungsten molecules in the target at 2000° K. is only about 10^{-6} times the velocity of light, and could therefore account for only a negligible part of the non-homogeneity of the x-rays. The velocity of the thermal motion of an electron, according to the principle of equipartition of energy, would give rise to a Doppler effect of the required order of magnitude; but the fact that the spectrum lines are characteristic of the tungsten

[1] This difference corresponds to a value of $\delta\theta_c > 3.2'$ in the first order. Thus by equation (1), $\delta x < c.\ 2700\lambda = 4.5 \times 10^{-5}$ cm.

atom indicates that it is not a free electron which emits the radiation, but one which is a part of the atom.

The damping of the electron's linear oscillations due to its own radiation, on the other hand, results in a width which is not negligible. The damping factor for a single oscillating electron is

$$\frac{4\pi^2}{3}\frac{e^2}{m\lambda^2 c},$$

where e, m and C have their usual significance. For $\lambda = 1.25$ Å.U., this means that the wave will be damped to $1/e$ of its amplitude in 3,400 vibrations. By an application of Fourier's integral, Professor Jauncey finds[1] that the effective width of the line due to this damping is about 0.06 minute of arc in the first order, which is 12 per cent. of the minimum width found experimentally.

It appears probable that the chief cause of the non-homogeneity observed in these experiments is that the lines themselves are complex. Thus the line $\lambda = 1.279$ is known to have a faint companion 0.008 Å.U. distant which was not separated in the first and second orders by the slits employed; and according to Sommerfeld's theory of the fine structure of spectrum lines, the line 1.242, on which most of the fourth order measurements were made, should have faint satellites at distances of about 0.0007, 0.002 and 0.006 Å.U.

The experiments described in this paper were performed at Palmer Physical Laboratory, Princeton University.

WASHINGTON UNIVERSITY,
SAINT LOUIS,
July 18, 1921.

[1] Cf. G. E. M. Jauncey elsewhere in this number of the PHYSICAL REVIEW.