Electron-Alkali-Metal Inelastic Recoil Experiments with Spin Analysis: Experimental Method and the Small-Angle Behavior of the $4^2S_{1/2} \rightarrow 4^2P_{1/2,3/2}$ Excitation of Potassium*

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The use of the atom-beam recoil technique in the study of differential inelastic electron-atom collisions with spin analysis is discussed. The general theory of the experimental method is presented. It is shown how to relate the observable R, the fractional depolarization of a polarized atom beam, to the several direct and exchange scattering amplitudes. A linear momentum-transfer analysis, required to connect the atomic and electron scattering angles, is also given. Some observations of the change of spin state of potassium, resulting from small-angle excitations of $4^2S_{1/2} \rightarrow 4^2P_{1/2,3/2}$ transitions are discussed, and are related to the threshold laws.

INTRODUCTION

This paper describes a novel experimental approach to the study of the inelastic scattering with spin analysis of atomic systems by low-energy electrons, based upon the atomic-beam recoil technique.[1] The measurements discussed here refer primarily to observations of the change of spin state of an alkali (potassium) beam resulting from the excitation transitions $4^2S_{1/2} \rightarrow 4^2P_{1/2,3/2}$ and subsequent decay back to the ground state. The experimental method, however, is a general one which is applicable, at least in principle, to the study of other spin-changing collisions involving other excited states, as a function of electron scattering angle and energy. The method is similar to that which we have already used to study elastic spin-exchange collisions in potassium.[1] In these crossed-beam experiments, the atomic beam is velocity and spin state selected before scattering and spin state analyzed after scattering. The experiments constitute *differential* measurements because the analyzer examines only a small range of atomic recoil scattering angles at a time. A suitable transformation obtained using linear momentum conservation then yields the corresponding range of electron polar scattering angles.

More detailed experimental results on both elastic and inelastic spin-change measurements will be presented in later papers. We intend to discuss here the general theory of the experimental method, particularly in connection with inelastic measurements, and to give some preliminary results on spin analysis of $4^2S_{1/2} \rightarrow 4^2P_{1/2,3/2}$ excitation of potassium.

The basic observable in this work is the quantity $R(\chi, \psi; v, V)$, the fractional depolarization of a polarized atomic beam of speed V which is recoil scattered into angles (χ, ψ) by unpolarized electrons of speed v. We will show here how R, measured as a function of the scattering parameters, can be related to the appropriate scattering amplitudes (actually, displacement of the beam in the plane of the detector, rather than the recoil angles, is used in the analysis). The relation of R for inelastic ($S \rightarrow P$) collisions to the polarization of resonance radiation produced by electron impact will be discussed. A linear momentum-transfer analysis, required in order to connect the atomic and electron scattering angles, is also presented.

THEORY

The details of the theory required for the analysis of this experiment are given in Appendix A, and we will present only the pertinent results here. We are interested in calculating the probability that an atom, initially prepared in a particular spin state will, after an excitation collision with an electron, return to the ground state with its spin orientation changed. In particular, consider the excitation of an alkali atom from a ground $^2S_{1/2}$ state to the first excited $^2P_{3/2,1/2}$ states. Assume that the scattering takes place in a laboratory magnetic field of sufficient strength to decouple the electronic and nuclear spins of the atom, thus allowing us to neglect hyperfine structure. We will also neglect the alteration of the spin orientation through a magnetic interaction during the collision. Finally, it is assumed that the system has initially been prepared with the spin of the atomic electron and the velocity of the incoming electron aligned with the external magnetic field.

Under the above assumptions, if the spin of the incoming electron is parallel to the magnetic field, the state ψ of the system immediately after the collision is proportional to (see Appendix A)

$$\psi \sim \{[f^0(\theta,\varphi)-g^0(\theta,\varphi)]\,Y_1^{\,0}+[f^1(\theta,\varphi)-g^1(\theta,\varphi)]\,(Y_1^{\,1}-e^{2i\varphi}Y_1^{\,-1})\}\,\alpha(1)\alpha(2). \tag{1}$$

If the spin of the incoming electron is antiparallel to the magnetic field, this state is proportional to

$$\psi \sim \{f^0(\theta,\varphi)Y_1^{\,0}+f^1(\theta,\varphi)(Y_1^{\,1}-e^{2i\varphi}Y_1^{\,-1})\}\,\alpha(1)\beta(2)$$
$$-\{g^0(\theta,\varphi)Y_1^{\,0}+g^1(\theta,\varphi)(Y_1^{\,1}-e^{2i\varphi}Y_1^{\,-1})\}\,\beta(1)\alpha(2). \tag{2}$$

In the above expressions $f^0(\theta,\varphi)$ and $f^1(\theta,\varphi)$ are the direct scattering amplitudes for excitation events in which the atom is left with its orbital angular momentum perpendicular and parallel to the velocity of the incoming electron, respectively.

These quantities are functions of the scattered electron's polar and azimuthal angles θ and φ, the polar axis being parallel to the direction of the velocity of the incoming electron. The g's are the corresponding amplitudes for exchange events. α and β represent spin-states parallel and antiparallel to the external field and 1 and 2 refer to the coordinates of the atomic and incoming electron, respectively. The Y's are the usual spherical harmonics referred to the atomic coordinates. The direct scattering amplitudes can be related to the elements of the T matrix by

$$f^{\mu}(\theta,\varphi)=\sum_{l,l'} Y_{l'}^{\,-\mu}(\theta,\varphi)\,C_{-\mu\,\mu\,0}^{\,l'\,1\,l}\,T_F(\{l',1\}l;\{l,0\}l), \tag{3}$$

where l and l' refer to the angular momentum of the incoming and scattered electron, respectively, and the C's are Clebsch-Gordan coefficients. There is, of course, a similar equation for the exchange amplitudes.

To calculate the fraction of atoms returning to the ground state with a particular spin orientation, Eqs. (1) and (2) are expanded in terms of $P_{1/2}$ and $P_{3/2}$ eigenfunctions.[2] Assuming that the lifetime of the excited state is long compared to $h/(E_{3/2}-E_{1/2})$, where $E_{3/2}-E_{1/2}$ is the energy difference of the $P_{3/2}$ and $P_{1/2}$ states, the relative decay rate to the ground state in question is calculated independently for the $P_{1/2}$ and $P_{3/2}$ states for each incoming spin orientation and the total decay rate is simply the sum of these. The fraction of atoms R returning to the ground state with spins antiparallel to the external field is given by

$$R=\frac{4}{9}+\frac{1}{9}\,\frac{6|g^1|^2+|g^0|^2-4|f^1-g^1|^2-4|f^1|^2}{|f^0-g^0|^2+|g^0|^2+|f^0|^2+2(|f^1-g^1|^2+|g^1|^2+|f^1|^2)}\;. \tag{4}$$

If the exchange amplitudes can be neglected then

$$R(\text{no exchange})=\tfrac{4}{9}-\tfrac{4}{9}\,|f^1|^2/(2|f^1|^2+|f^0|^2)\;. \tag{5}$$

Alternatively, Eqs. (4) and (5) can be written as

$$R=\tfrac{4}{9}+\tfrac{1}{18}\,(|g^0|^2+10|g^1|^2-8\sigma_1)/\sigma \tag{6}$$

$$\text{and}\quad R(\text{no exchange})=(\sigma_0+\sigma_1)/\sigma\;, \tag{7}$$

where $\sigma=\sigma_0+2\sigma_1$ and σ_0,σ_1 are the differential cross sections for excitation with $\Delta m=0,\pm1$, respectively, i.e.,

$$\sigma_{0,1}=\tfrac{3}{4}|f^{0,1}-g^{0,1}|^2+\tfrac{1}{4}|f^{0,1}+g^{0,1}|^2\;. \tag{8}$$

The interesting problem of the behavior of the various scattering amplitudes in the threshold and the high-energy limits has received considerable attention recently.[3] Using simple angular-momentum conservation arguments, and neglecting spin-dependent forces, it can be shown[4] that f^1/f^0 and g^1/g^0 vanish at threshold while f^0/f^1, g^0, and g^1 vanish at high energies for all finite scattering angles. Thus the limiting values of R are

$$R(\text{threshold}) = \tfrac{4}{9} + \tfrac{1}{18}\,\sigma_{0\text{ex}}/\sigma_0 \tag{9}$$

and $$R(\text{high energy}) = \tfrac{2}{9}\,, \tag{10}$$

where $\sigma_{0\text{ex}} = |g^0|^2$.

If $\sigma_{0\text{ex}}$ can be neglected, then

$$R(\text{threshold, no exchange}) = \tfrac{4}{9}\ . \tag{11}$$

It is of interest to compare our experiment with those in which polarization of resonance radiation is observed.[3,4] The normal procedure in that work is to measure the amount of linear polarization of the optical radiation of certain atomic transitions, which are excited by monoenergetic electrons, usually at an angle of 90° with respect to the direction of the initial momentum of the exciting electrons. In these measurements, only unpolarized electrons and atoms have been employed to date. If we consider the transition $({}^2S_{1/2} \rightarrow {}^2P_{1/2,\,3/2})$, the percent polarization can then be shown to be

$$P = 300\,\frac{(|F^0 - G^0|^2 + |F^0|^2 + |G^0|^2) - (|F^1 - G^1|^2 + |F^1|^2 + |G^1|^2)}{7(|F^0 - G^0|^2 + |F^0|^2 + |G^0|^2) + 11(|F^1 - G^1|^2 + |F^1|^2 + |G^1|^2)}\ , \tag{12}$$

where

$$P = 100(I_{\|} - I_{\perp})/(I_{\|} + I_{\perp})$$

and $I_{\|}$ and $I_{\perp}$ are the radiation intensities with polarization parallel and perpendicular to the exciting electron's initial momentum. The quantities F and G are the integral forms of the various scattering amplitudes, e.g.,

$$F^0 = 2\pi\int_0^{\pi} f^0(\theta)\sin\theta\, d\theta\ ,$$

so that Eq. (12) can be written as

$$P = 300\,(\sigma_T{}^0 - \sigma_T{}^1)/(7\sigma_T{}^0 + 11\sigma_T{}^1)\ , \tag{13}$$

where $\sigma_T{}^0$, $\sigma_T{}^1$ are the effective integral (total) cross sections for transitions with $\Delta m_l = 0, \pm 1$, respectively. It is important to note that in the optical formulae, only the total cross sections for excitation with $\Delta m_l = 0, \pm 1$ are involved, that is, one cannot separately distinguish the exchange contribution. This statement is equally true when observing angles other than 90°, although Eqs. (12) and (13) must then be modified. Thus the threshold law for 90° observation, and the high-energy limit, are,

$$P(\text{threshold}) = \tfrac{3}{7}\ ,\quad \text{and } P(\text{high energy}) = -\tfrac{3}{11}\ ,$$

respectively, independent of the role played by exchange in the excitation process.

It is seen that the recoil-type experiment with spin analysis differs from, and is superior to, the optical experiment in that the former refers to differential and the latter to integral cross sections, i.e., recoil studies can be made of changes of spin state as a function of electron scattering angle. A second difference lies in the fact that the recoil analysis depends upon exchange effects, which is not the case in the optical experiment.

EXPERIMENTAL METHOD

The experimental method consists of cross firing a velocity- and state-selected atomic beam by an electron beam, and observing the spin-analyzed angular distribution pattern of the inelastically scattered atoms, noting that they have, of course, decayed to the ground state long before they reach the analyzer.

Our measurements yield $R(\chi, \psi; v, V)$ since they consist of observations of the ratio of spin changed to total scattering signals at a given atom scattering angle as a function of electron energy. Of course, the spreads in electron and atom beam speeds, as well as the finite widths and varying trajectories of both beams, result in the measured R being actually an average over a range of scattering angles and energies.

The apparatus consists of an alkali oven source, a Stern-Gerlach velocity selector and polarizer,[5] a scattering region, an E-H gradient-balance analyzer,[6] and a surface ionization detector. A sketch of the experimental arrangement is shown in Fig. 1. For potassium, the magnetic fields employed in the selector and analyzer magnets and in the scattering chamber are sufficiently high so that electron and nuclear spins are decoupled. The potassium ground state can then be described in terms of pure $M_J = \pm\frac{1}{2}$ states.

In normal operation, the alkali oven is offset from the beam axis. The Stern-Gerlach magnet thereby causes one of the spin states to pass through the apparatus, and velocity-selects this state with a resolution $\Delta V/V$ (where V is the atomic speed) of approximately 10% at the same time. The E-H gradient-balance magnet (analyzer) is adjusted so as to pass only those atoms which have changed their spin state. It is a characteristic of the E-H gradient-balance magnet that balanced atoms are transmitted with no significant alteration in their trajectories. That is, this type of analyzer acts as a "spin filter," passing the desired state without significant distortion of the particle trajectories while rejecting all others.[6] The polarizer-analyzer combination employed in this experiment is about 99% efficient, that is, only about 1% of the unwanted spin state arrives at the detector position with both magnets operating. The ability of the analyzer to transmit the scattered beam without distortion of the atom trajectories is desirable because it simplifies the transformation from atomic to electron scattering angles. Most of the atoms in the unwanted spin state strike the polarizer pole faces or polarizer collimating slit. The beam entrance channel to the interaction region of the scattering gun serves as a second collimator, for the polarizer-velocity selector. Only an estimated 2% of the unwanted spin state can enter into the interaction region and be possibly recoil scattered into the detector. The resulting error in R caused by this impurity is also of this same order.

The scattering gun is modulated at a frequency of 10 cps and a phase-sensitive lock-in detector is employed. The output of the lock-in can be digitalized and integrated by using a Vidar voltage-to-frequency converter and a scalar. Normally, however, the lock-in output is directly displayed on an x-y recorder, whose abscissa is proportional to the detector position. The detector is automatically driven (see Fig. 1) and its position accurately determined by a calibrated linear potentiometer. A typical scattering curve is shown in Fig. 2, in which the spin filter is not employed. In the figure, taken at 5 eV, the direction of the incident electrons is to the left. The negative signal (region A) represents scattering out from the forward beam (shown in re-

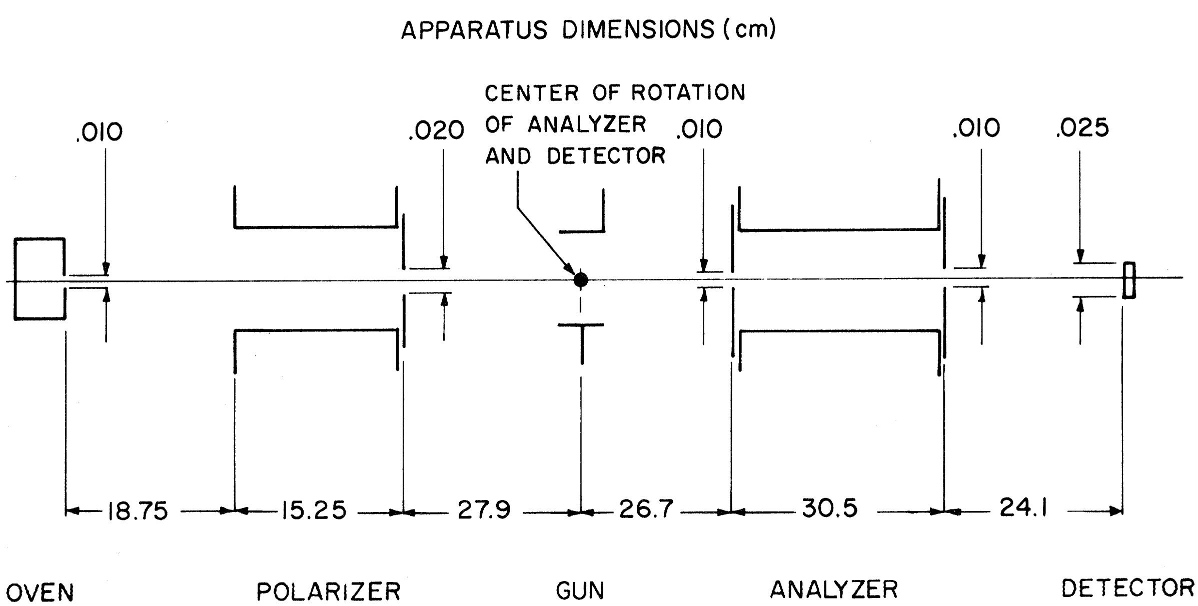

FIG. 1. Experimental arrangement and apparatus dimensions. The analyzer and detector assemblies can be automatically and independently driven, and rotate about the scattering center.

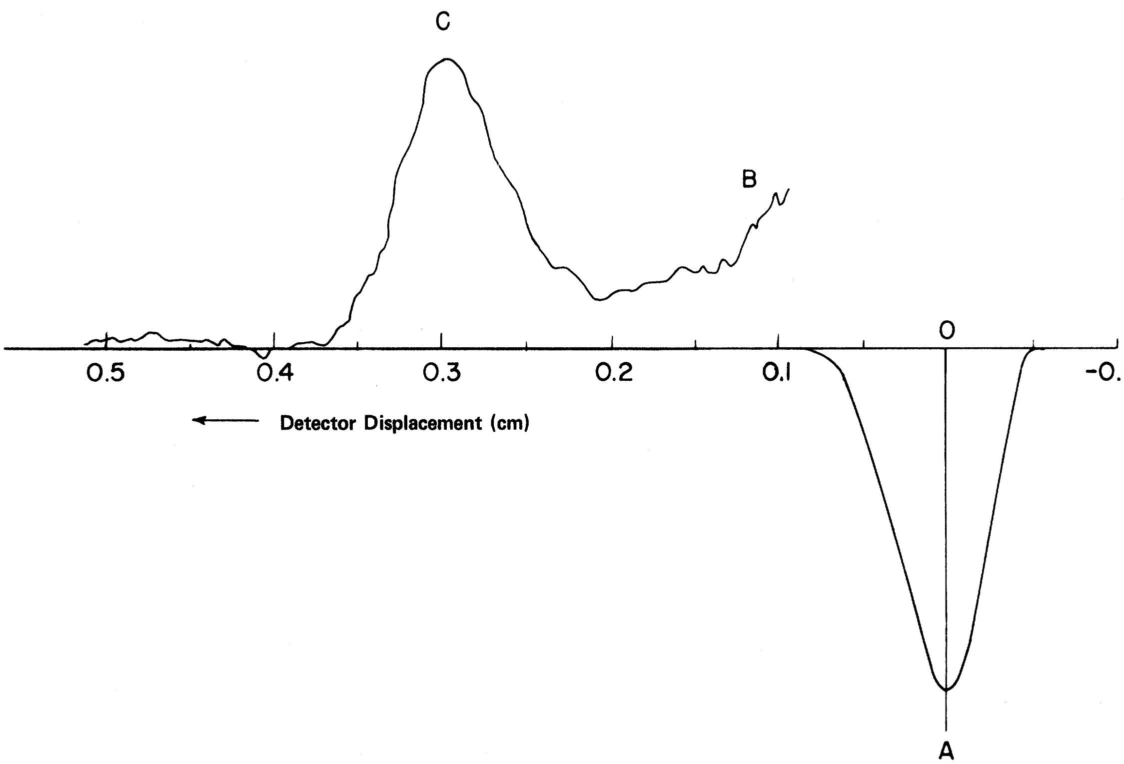

FIG. 2. Raw differential-scattering data, taken at 5 eV. The direction of the incident electrons (modulated at 10 cps) is to the left. Region A shows scattering out and approximately duplicates the dc beam shape. It is shown at considerably reduced gain. Region B contains only elastically scattered atoms, and Region C contains inelastically scattered atoms, primarily $4^2P_{1/2} \to 4^2P_{1/2,3/2}$ at small (polar) scattering angles. Atoms scattered by electrons whose polar scattering angle is 90° would appear at 2.07 cm for all channels (elastic and inelastic).

duced gain). Its shape is approximately the same as that of the unscattered beam, although there is some distortion in the left-hand portion due to contribution to the signal of scattered-in particles from the right-hand portion of the beam. The positive portion of the signal is due to scattering in. The particles in region B are elastically scattered. The particles in region C are predominantly inelastically scattered at small (electron) scattering angles (${}^4S_{1/2} \to {}^4P_{1/2,3/2}$ transition). This angular discrimination between elastic and inelastic scattering arises from simple considerations of conservation of momentum and energy.

A detailed momentum-transfer analysis is prescribed in Appendix B. It is shown there [Eq. (B-13)] that ΔN_S, the scattered atom current reaching the detector is given by

$$\Delta N_S = \frac{2w}{HL}\sum_i \int_{\varphi_1}^{\varphi_2} d\varphi\,\sigma_i(\theta) I_e(E) \times J_a(x_0', z_0'; V)\,dE\,dx_0'\,dz_0'\,dV/\beta_i V, \tag{14}$$

where φ_2, φ_1 are the limiting azimuthal scattering angles which a detector of finite height h can collect. This is the general equation relating recoil-scattering signal at a detector of height h and width w at a displacement z from the beam axis to the beam currents, shapes, and velocity distributions. It should be noted that the relation between electron and atom scattering angles [Eqs. (B-10), (B-11)] depend upon electron energy loss and there is thus a separate $\theta - \psi$ relation for each scattering channel. Thus Eq. (14) includes a summation over all relevant inelastic (as well as elastic) collision channels.

Equation (14) will now be reduced by the use of simplifying assumptions applicable to the present experiment. First, it is assumed that $\chi_0 \ll h/2$ for all points in the dc beam, i.e., that the dc beam height is small compared to the detector height. The azimuthal form factor (AFF) (see Appendix B) can then be approximated by

$$\gamma_i(\theta, V, E) \approx \frac{2}{\pi}\sin^{-1}\frac{h}{2L\beta_i \sin\theta}\ . \tag{15}$$

The scattering signal is then

$$\Delta N_s = \frac{4\pi w}{HL}\sum_i \gamma_i(\theta, V, E)\sigma_i(\theta)I_e(E)$$

$$\times J_a(x_0', z_0', V)dE dx_0' dz_0' dV/\beta_i V, \qquad (16)$$

where it is understood that θ for each channel i can be obtained as a function of $z - z_0'$.

We now define the total atom-beam current per unit length along the detector at z_0' as

$$I_a(z_0'; V) = \int J_a(x_0', z_0'; V)dx_0' , \qquad (17)$$

so that

$$\Delta N_s = \frac{4\pi w}{HL}\sum_i \int \gamma_i(\theta, V, E)\sigma_i(\theta)I_e(E)$$

$$\times I_a(z_0'; V)dE dz_0' dV/\beta_i V . \qquad (18)$$

In general, of course, the amount of unfolding required to extract $\sigma_i(\theta)$ from Eq. (18) will depend upon the "quality" of the experiment, for example, upon the degrees of monochromaticity and parallelism of the beams. For a "pencil" atom beam (negligible cross-sectional area), assumed to apply here,

$$\Delta N_s = \frac{4\pi w}{HL}\sum_i \sigma_i(\theta)\int \gamma_i(\theta, V, E)$$

$$\times I_e(E)I_a(V)dE dV/\beta_i V , \qquad (19)$$

where $I_a(V)$ is the total dc atom-beam current.

EXPERIMENTAL DATA

The data presented here are intended to be illustrative of the general principles discussed in the previous sections. The momentum-transfer equations will be employed in their simplest form, i.e., we will assume that the "pencil" beam approximation is valid,[7] so that Eq. (19) obtains. The AFF is a sharply peaked function of θ with maxima at 0° and 180°, and as a result it is expected that ΔN_S will be sharply peaked at z corresponding to 0° scattering. There should also be a second peak in ΔN_S corresponding to 180° scattering. However, this will be much smaller than the 0° peak, first, because $\sigma(180°)$ is usually much smaller than $\sigma(0°)$, and second, because the finite velocity distribution in both beams produces a larger dispersion in ΔN_S at 180° than at 0°. Figure 3 illustrates these points, referring to calculated AFF's for elastic scattering in potassium at 1 eV (inelastic AFF's would be similar, with reduced abscissae). Figure 3a shows a plot of the elastic AFF for monoenergetic atom and electron beams, using our apparatus parameters. Figure 3b shows a plot of the elastic AFF averaged over the measured electron energy distribution. It can be seen that the large peak at 0° is unaffected by the spread in electron energies, but that the 180° peak is smeared out.

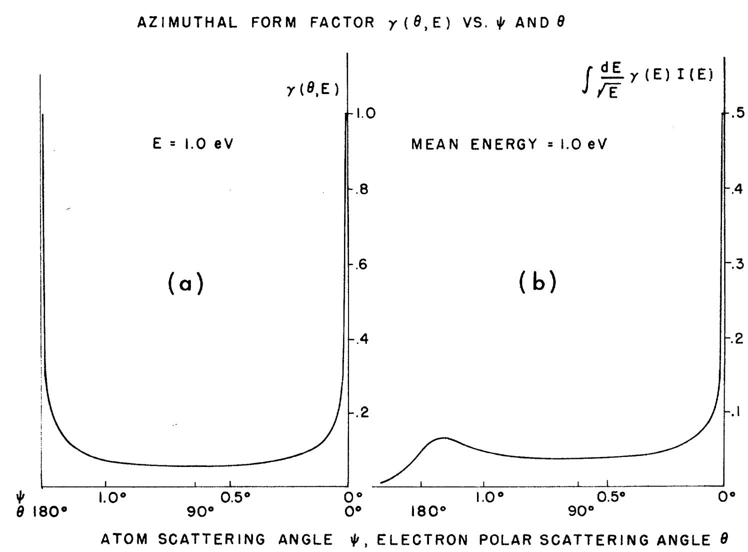

FIG. 3. Calculated azimuthal form factors (AFF), for elastic scattering of a monoenergetic potassium beam by 1-eV electrons (atom speed = 700 m/sec). In Fig. 3a, the electron beam is also monoenergetic. In Fig. 3b, the experimentally determined electron energy distribution is folded in, to obtain an "effective" AFF, which is reduced and spread out at large angles. The upper abscissa scale refers to ψ, the lower to θ. In Fig. 3b, the θ scale refers to the mean electron energy of 1 eV.

For inelastic scattering in potassium we can expect the principal contribution in Eq. (19) to come from the resonance transition $4S \rightarrow 4P$, since the excitation cross section for this reaction will be substantially larger than those to other states.[8] Then the sum in Eq. (19) reduces to a single term involving the resonance transition only.

In practice, since γ_i is so sharp at $\theta = 0$, it can be treated as a δ function, and so for small-angle scattering ΔN_S will reproduce the dc beam fairly closely. This can be seen by referring to Fig. 2, where region C represents small-angle resonance scattering, which indeed possesses a profile similar to that of the dc beam (region A). This point is further illustrated by the data in Fig. 4, which shows small angle $4S \rightarrow 4P$ excitations at several energies between 3 and 15 eV. The arrows indicate the calculated positions of $\gamma(\theta = 0°)$ for $4S \rightarrow 4P$.

By now placing the detector at the peak of ΔN_S and measuring R we obtain a determination of this quantity, averaged over the small angles which are involved in measurements. This angle range arises primarily from the contribution of small-angle scattering in from the right-hand portion of the dc beam (Figs. 2, 4). It is difficult to quantitatively establish the appropriate average. We will assume that $\theta \approx 0°$, although some events caused by scattering with polar angles of up to 10° are actually included.[9]

Figure 5 shows measurements made of R as a function of θ, assuming only $4S \rightarrow 4P$, for energies of between 1.6 (threshold) and 15 eV. The electron energy spread in this work was 250 MeV full width at half-maximum (FWHM). Certain features are apparent from these measurements. It can be seen that, except at energies very close to threshold, R falls within the upper and lower bounds permitted by conservation of momentum arguments.[10] The threshold value is close to the predicted $\frac{4}{9}$ value, which obtains without exchange. However, very close to threshold R exceeds the nonexchange value. A series of measurements at threshold give here

$$R(\theta \approx 0°,\ \text{threshold}) = 0.474 \pm 0.014\ ,$$

where the quoted error is due solely to statistics. This value requires [see Eq. (6)]

$$\frac{|g_0|^2}{\sigma_0}\ (\theta \approx 0°, \text{threshold}) = 0.54 \pm 0.25\ .$$

At 15 eV, $R = 0.324 \pm 0.004$.

CONCLUSION

This paper has discussed the atomic-beam recoil technique, especially as applied to the $4S$-$4P$

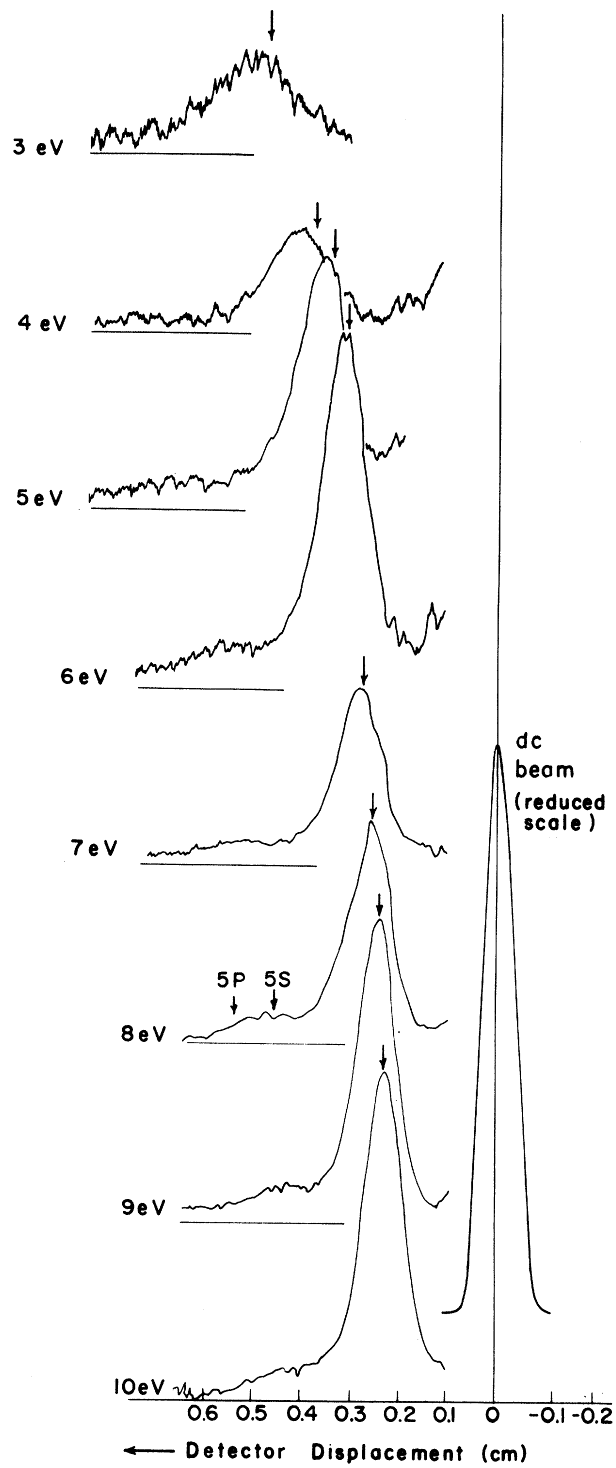

FIG. 4. Inelastic-scattering data at energies of 3, 4, 5, 6, 7, 8, 9 and 10 eV for potassium. Arrows indicate the calculated positions for the appearance of the $4S$-$4P$ transitions, $\theta = 0°$. For comparison, the $4S$-$5P$ and $4S$-$5S$ transitions ($\theta = 0°$) would appear at the indicated positions in the 8-eV curve.

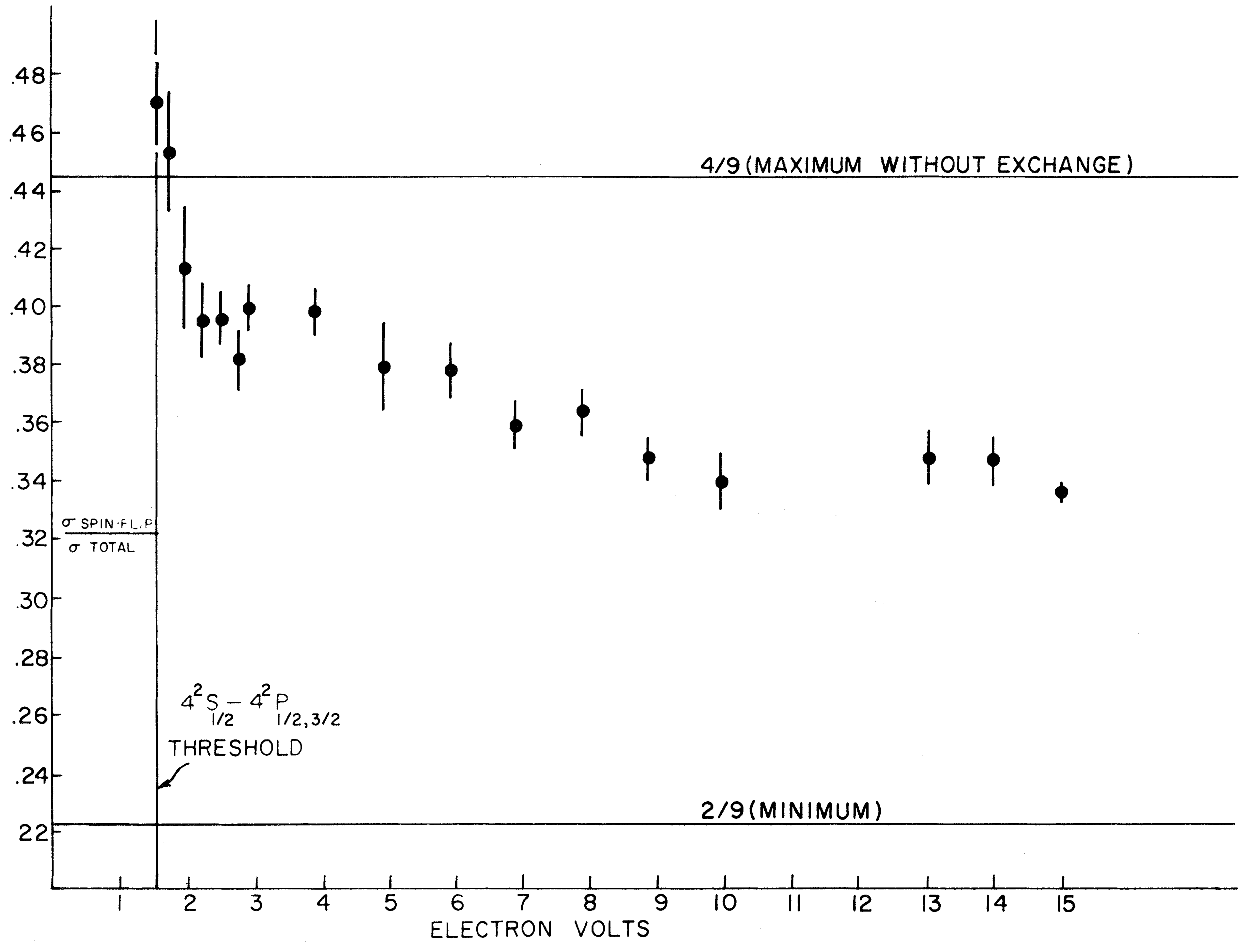

FIG. 5. $R(\theta \approx 0°)$ (ratio of spin flip to full differential cross section) versus electron energy, for $4S$–$4P$ excitation of potassium. The upper horizontal line represents the maximum theoretically allowable value of R, neglecting exchange. The lower horizontal line represents the minimum theoretically allowable value of R. This value should be approached in the high-energy limit.

excitation in potassium. We have shown how to relate the observable R to the various direct and exchange scattering amplitudes, and we present a momentum-transfer analysis appropriate to inelastic collisions (Appendix B). Data have been presented on the determination of $R(\theta)$ for $\theta \approx 0°$ (electron polar scattering angle).

This work is now being extended to the measurement of $R(\theta)$ at larger angles, using a scattering gun which rotates about the interaction volume. This effectively enables us to observe scattering out of the plane, which can be uniquely identified with $4S$-$4P$ scattering as a function of scattering angle.

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APPENDIX A.

Relating the Observables to Scattering Amplitudes

Consider the following experimental arrangement. A beam of atoms is intersected at right angles by a beam of electrons. The scattering region is in a magnetic field of sufficient strength to decouple the nuclear and atomic spins so that hyperfine effects can be neglected. The direction of this external field is parallel to that of the momentum of the incoming electron. It is further assumed that the collision time is sufficiently small to enable us to neglect magnetic interactions during the collision. We wish to derive an expression for the fraction of electrons scattered into a particular set of angles after an inelastic collision, in which we specify the polarization of the scattered electron, the hyperfine state of the atom after returning to the ground state, and the polarization of the emitted photon, assuming that the initial polarization of the incoming electron and the initial hyperfine state of the atom are known. In particular the excitation of an alkali atom from a ground $^2S_{1/2}$ state to the first excited $P_{1/2,\,3/2}$ states is considered.

We will first neglect exchange effects, including them at a later point. If both magnetic interactions during the collision and exchange effects are neglected the spin of the incoming electron will play no role, and observation of the spin of the scattered electron will add no new information. Knowledge of the spin state of the atom, however, both before and after the collision is important. Immediately after the collision the spin of the atomic electron will have the same orientation it had immediately before the collision. Just after the collision the wave function of the atom will be proportional to

$$\Psi \sim \sum_\mu f^\mu Y_1^{\,\mu} \lambda_{m_s} , \tag{A-1}$$

where f^μ is the direct scattering amplitude, a function of the electron polar scattering angle θ and azimuthal angle φ, for an excitation event in which the z component of the atomic orbital angular momentum has the quantum number μ. Here the z direction is taken as the direction of the external magnetic field, and since we are considering an excitation event to a P state, μ can have the values $1, 0, -1$. $Y_1^{\,\mu}$ is the usual spherical harmonic as a function of the atomic coordinates and λ_{m_S} represents the spin state of the atom.

For an S-P excitation there are two independent scattering amplitudes in Eq. (A-1), f^1 and f^0, since it can be shown from parity considerations that

$$e^{i\varphi} f^{1} = -e^{-i\varphi} f^{-1} \quad .$$

Using the expansions

$$Y_1^{\,\mu} \lambda_{m_s} = \sum_j C(1, \tfrac{1}{2}, j; \mu, m_s, \mu + m_s) \Psi_j^{\,\mu + m_s} \tag{A-2}$$

and
$$\Psi_j^{\,\mu + m_s} = \sum_{M_S} C(1, \tfrac{1}{2}, j; \mu + m_s - M_s, M_s, \mu + m_s)\, Y_1^{\,\mu + M_S - M_S} \lambda_{M_S} , \tag{A-3}$$

where the C's are Clebsch-Gordan coefficients, in Eq. (A-1) and including a time factor, the wave function of the excited atom at any time t after the collision is

$$\Psi \sim \sum_{j \mu M_s} f^\mu C(1, \tfrac{1}{2}, j; \mu, m_s, \mu + m_s) C(1, \tfrac{1}{2}, j; \mu + m_s - M_s, M_s, \mu + m_s)$$
$$\times Y_1^{\,\mu + m_s - M_S} \lambda_{m_s} e^{-iE(j,\, \mu + m_s)t/\hbar} \quad . \tag{A-4}$$

We can now calculate the probability that the atom will decay to a particular ground state specified by M_S, with the emission of a photon polarized either parallel or perpendicular to the z direction. If it is assumed that the probability of an atom decaying after a time t during the time interval dt is given by the square of the transition matrix element times $e^{-t/\Gamma} dt/\Gamma$, where Γ is the lifetime of the state, we can, by integration from $t=0$ to $t=\infty$, obtain the transition probability after a sufficiently long time. The above procedure leads to the result representing the relative probability P that the atom returns to the ground state with spin component M_S, while the electron is scattered into the angles θ, φ, and a photon of specified polarization is emitted into the angles A, B

$$P \sim \sum_{j\mu} |f^\mu|^2 \left| C(1, \tfrac{1}{2}, j; \mu, m_s, \mu + m_s)\right|^2 \left| C(1, \tfrac{1}{2}, j; \mu + m_s - M_s, M_s, \mu + m_s)\right|^2$$

$$\times[(\delta_{\mu+m_s-M_s,\pm1}\tfrac{1}{2}\cos^2A+\delta_{\mu+m_s-M_s,0}\sin^2A)_{\parallel}+\tfrac{1}{2}\delta_{\mu+m_s-M_s,\pm1\perp}]$$

$$+\sum_{\substack{jj'\\ \mu\neq\mu'}}[C(1,\tfrac{1}{2},j;\mu,m_s,\mu+m_s)C(1,\tfrac{1}{2},j';\mu',m_s,\mu'+m_s)$$

$$\times C(1,\tfrac{1}{2},j;\mu+m_s-M_s,M_s,\mu+m_s)C(1,\tfrac{1}{2},j';\mu'+m_s-M_s,M_s,\mu'+m_s)]$$

$$\times|f^{\mu}|\,|f^{\mu'}|\operatorname{Re}\left[\left(\frac{1-i\eta(j,\mu+1/2;j',\mu'+1/2)}{1+\eta^2}\right)e^{-i\Delta_f(\mu,\mu')}\right.$$

$$\times\{[e^{2i(b-\varphi)}\tfrac{1}{2}\cos^2A\,\delta_{\mu+m_s-M_s+1}\delta_{\mu-\mu',+2}-\sin A\cos A\,e^{\pm i(B-\varphi)}\delta_{\mu+m_s-M_s,\pm1}\delta_{\mu-\mu',\pm1}]_{\parallel}$$

$$\left.-[\tfrac{1}{2}e^{2i(B-\varphi)}\delta_{\mu+m_s,+1}\delta_{\mu-\mu',+2}]_{\perp}\}\right], \tag{A-5}$$

where $\eta(j,m;j',m')=[E(j,m)-E(j',m')]\Gamma/\hbar$ (A-6)

and $\Delta_f(\mu,\mu')$ is the constant (i.e., independent of φ) phase difference between f^{μ} and $f^{\mu'}$.

Consider first the set of terms in the second summation. These terms involve the phase difference $\Delta_f(\mu,\mu')$ between the scattering amplitudes f^0 and f^1. Observation of these terms could lead to a measurement of this phase factor. The observability of these terms depends upon the magnitude of η. Since η is a function of the energy separation of two levels we must have levels close enough together for these terms to play a role. They represent an interference effect in the decay due to the fact that the atom is simultaneously decaying from two different states. These are the same terms appearing in level-crossing experiments. In our case, to observe this interference it is necessary to make a simultaneous observation of the scattered electron and photon since if we average over either φ or B the interference terms disappear. It is clear that this should happen since B and φ have experimental significance only in terms of the relative angle between the emitted photon and the recoiling electron. In the present experiment we have not performed the necessary simultaneous observations and we will therefore neglect these terms in the rest of our analysis. It should however be kept in mind that observation of these interference terms makes it possible to obtain the phase factors between the various scattering amplitudes.

We now generalize these results by including exchange effects. Let α,β denote spin-functions parallel and antiparallel to the external field, respectively, and 1, 2 refer to incoming and atomic electrons, respectively. If the incoming electron is in an α state, the full wave function immediately after the collision is

$$\Psi\sim\sum_{\mu}f^{\mu}Y_1^{\mu}\lambda_{m_s}(2)\alpha(1)-\sum_{\mu}g^{\mu}Y_1^{\mu}\lambda_{m_s}(1)\alpha(2)\,, \tag{A-7}$$

where the f^{μ} are again the direct scattering amplitudes and the g^{μ} are the corresponding exchange amplitudes. If the atom is initially polarized with $m_s=\frac{1}{2}$ then $\lambda_{m_s}=\alpha$ and

$$\Psi\sim\sum(f^{\mu}-g^{\mu})Y_1^{\mu}\alpha(2)\alpha(1). \tag{A-8}$$

For this situation the various transition probabilities can be obtained from the previous results by substituting $(f^{\mu}-g^{\mu})$ for f^{μ} in Eq. (A-5) and letting $m_s=+\frac{1}{2}$.

If the incoming electron is in a β state the wave function immediately after the collision is

$$\Psi\sim\sum_{\mu}f^{\mu}Y_1^{\mu}\lambda_{m_s}(2)\beta(1)-\sum_{\mu}g^{\mu}Y_1^{\mu}\lambda_{m_s}(1)\beta(2)\,. \tag{A-9}$$

Again, if $m_s=+\frac{1}{2}$,

$$\Psi \sim \sum_{\mu} f^{\mu} Y_1^{\mu} \alpha(2)\beta(1) - \sum_{\mu} g^{\mu} Y_1^{\mu} \alpha(1)\beta(2)\,. \tag{A-10}$$

The expressions of the first and second summation in Eq. (A-10) are obtained from Eq. (A-5), with $m_s = +\frac{1}{2}, -\frac{1}{2}$, respectively.

Table I summarizes the results obtained using the above procedure, assuming that the initial state of the atom is given by $m_s = +\frac{1}{2}$.

Thus far only two types of experiments have been performed, both involving unpolarized electrons. Either the polarization of the emitted radiation is observed at some particular angle with no observation made of the initial or final spin state of the atom, or the atom is initially in a known spin state with observations made of the final spin state of the atom as a function of the atom recoil angle, with no observation of the emitted radiation. In both cases since the electrons are initially unpolarized with no observation made of the final spin of the electron, we must therefore sum the results over the initial and final spin states of the electron. In the first case we must also sum over M_S and average over all electron scattering angles. This leads to the results that the photon polarization probability is proportional to:

parallel $$[\tfrac{5}{9}(|F^0|^2 + |G^0|^2 + |F^0 - G^0|^2 + \tfrac{4}{9}(|F^1|^2 + |G^1|^2 + |F^1 - G^1|^2)]\sin^2 A$$

$$+[\tfrac{2}{9}(|F^0|^2 + |G^0|^2 + |F^0 - G^0|^2 + \tfrac{7}{9}(|F^1|^2 + |G^1|^2 + |F^1 - G^1|^2)]\cos^2 A$$

perpendicular $$\tfrac{2}{9}(|F^0|^2 + |G^0|^2 + |F^0 - G^0|^2) + \tfrac{7}{9}(|F^1|^2 + |G^1|^2 + |F^1 - G^1|^2)\,,$$

TABLE I. Tabulation of final electron and atom spin-states, assuming initial atomic spin state $m_s = +\frac{1}{2}$ and initial electron spin-states parallel (α) and antiparallel (β) to external magnetic field. $S \to P$ excitation in alkali (nuclear spin neglected).

Inital spin state of incoming electron	Final spin state of atom M_S	Final spin state of outgoing electron	Probability of achieving specified final state with the emission of photon is proportional to: (photon parallel to final spin state)	(photon perpendicular to final spin state)
α	$+\frac{1}{2}$	α	$\frac{5}{9}\lvert f^0 - g^0\rvert^2 \sin^2 A + \frac{7}{9}\lvert f^1 - g^1\rvert^2 \cos^2 A$	$\frac{7}{9}\lvert f^1 - g^1\rvert^2$
α	$+\frac{1}{2}$	β	0	0
α	$-\frac{1}{2}$	α	$\frac{2}{9}\lvert f^0 - g^0\rvert^2 \cos^2 A + \frac{4}{9}\lvert f^1 - g^1\rvert^2 \sin^2 A$	$\frac{2}{9}\lvert f^0 - g^0\rvert^2$
α	$-\frac{1}{2}$	β	0	0
β	$+\frac{1}{2}$	α	$\frac{4}{9}\lvert g^1\rvert^2 \sin^2 A + \frac{2}{9}\lvert g^0\rvert^2 \cos^2 A$	$\frac{2}{9}\lvert g^0\rvert^2$
β	$+\frac{1}{2}$	β	$\frac{5}{9}\lvert f^0\rvert^2 \sin^2 A + \frac{7}{9}\lvert f^1\rvert^2 \cos^2 A$	$\frac{7}{9}\lvert g^1\rvert^2$
β	$-\frac{1}{2}$	α	$\frac{5}{9}\lvert g^0\rvert^2 \sin^2 A + \frac{7}{9}\lvert g^1\rvert^2 \cos^2 A$	$\frac{7}{9}\lvert g^1\rvert^2$
β	$-\frac{1}{2}$	β	$\frac{2}{9}\lvert f^0\rvert^2 \cos^2 A + \frac{4}{9}\lvert f^1\rvert^2 \sin^2 A$	$\frac{2}{9}\lvert f^0\rvert^2$

where $F^{\mu} = 2\pi\int_0^{\pi} f^0(\theta)\sin\theta d\theta$. For $A = 90°$, we obtain the results of Eqs. (12), (13).

In the second case a summation is performed over the polarization of the photon and averaged over all photon directions. This leads to the following results:

M_S Probability proportional to:

$$+\tfrac{1}{2}\quad 20(|f^0|^2 + |g^0|^2 + |f^0 - g^0|)^2 - 4|g^0|^2 + 56(|f^1|^2 + |g^1|^2 + |f^1 - g^1|^2) - 40|g^1|^2$$

$$-\tfrac{1}{2}\quad 16(|f^0|^2 + |g^0|^2 + |f^0 - g^0|^2) + 4|g^0|^2 + 16(|f^1|^2 + |g^1|^2 + |f^1 - g^1|^2 + 40|g^1|^2\,.$$

The ratio of the number of atoms returning to the $M_S = \frac{1}{2}$ state to the total number scattered for a particular set of recoil angles is given by

$$R = \tfrac{1}{9}\left(4\sigma^0 + 4\sigma^1 + \tfrac{1}{2}|g^0|^2 + 5|g^1|^2\right)/(\sigma^0 + 2\sigma^1)\ , \tag{A-11}$$

where $2\sigma^\mu = |f^\mu|^2 + |g^\mu|^2 + |f^\mu - g^\mu|^2$.

This relation can be rewritten to obtain Eqs. (4), (6).

APPENDIX B. MOMENTUM-TRANSFER ANALYSIS

Consider the momentum-transfer problem for an atomic beam which is cross fired at right angles by an electron beam. The electron beam travels in the $+z$ direction while the atoms travel within a small cone of angles about the y axis (see Fig. 6). The coordinates of an unscattered atom in the interaction region are (x_0, y_0, z_0) and in the plane of the detector are (x', y', z'), while the coordinates of a scattered atom in the detector plane are (x, y, z). The direction of the velocity of an unscattered atom is specified by the angles (ψ_0, χ_0) while that of a scattered atom is specified by (ψ, χ). Here ψ is the angle between the projection of the atomic velocity in the y-z plane and the y axis, and χ is the angle between the atomic velocity and its projection in the y-z plane. (When $\psi_0, \chi_0 \ll \psi, \chi$, then ψ, χ are simply the scattering angles in the plane of scattering and perpendicular to the plane of scattering, respectively.)

Consider the interaction of the electron and atom beams within an element of volume $dx_0 dy_0 dz_0$. The electron-number current density within this region for electrons whose energies are in the range dE is $J_e(x_0, y_0, z_0; E)dE$, while the number density of atoms in this region with velocities lying within the angular range $d\psi_0, dx_0$ and possessing speeds in the range dV is

$$M_a(x_0, y_0, z_0; \psi_0, \chi_0, V)\, d\psi_0 d\chi_0 dV\ .$$

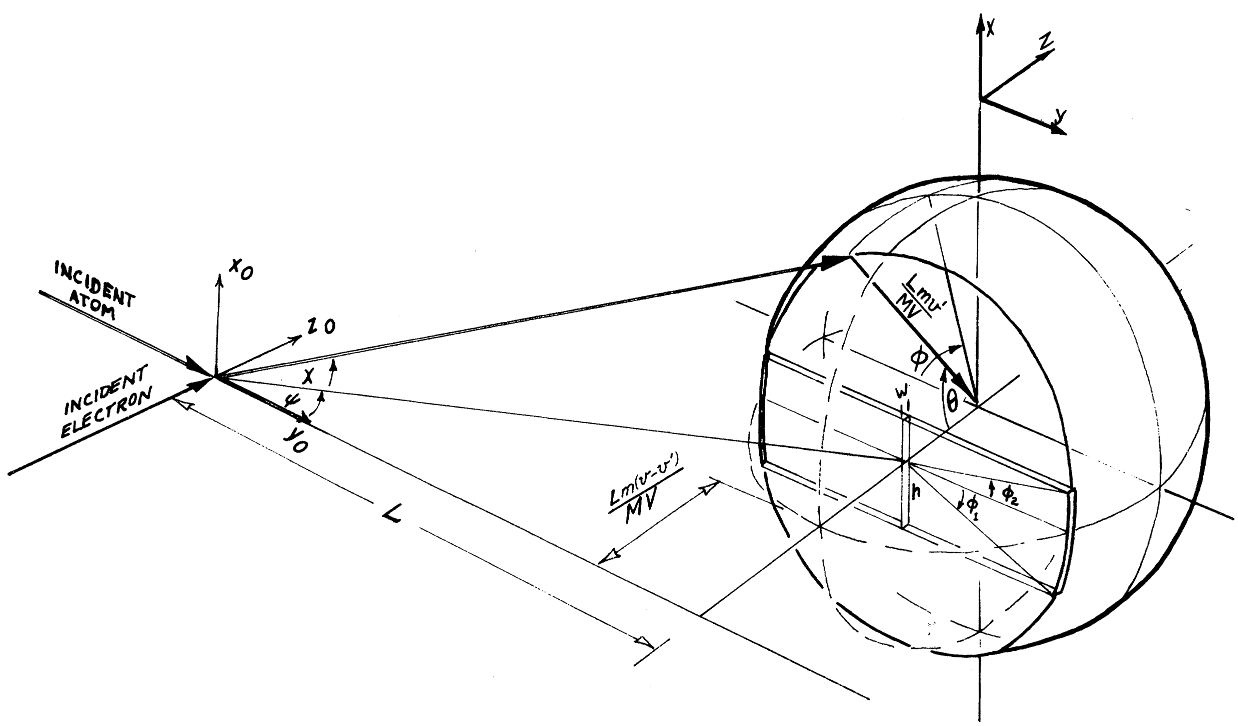

FIG. 6. Momentum diagram for simplified electron-atom beam system, in which the incident electron and atom beams are traveling along the z_0 and y_0 axes, respectively.

The number of atoms scattered from the element dx_0, dy_0, dz_0 into the angular range $d\psi, d\chi$ possessing speeds in the range dV is

$$dN_s = \sum_i J_e(x_0, y_0, z_0; E)\, dE dx_0 dy_0 M_a(x_0, y_0, z_0; \psi_0, \chi_0, V)\, d\psi_0 d\chi_0 dV \sigma_i'(\psi - \psi_0, \chi - \chi_0; E, V) d\psi d\chi dz_0, \quad \text{(B-1)}$$

where $\sigma_i' \, d\psi d\chi$ is the probability of scattering an atom of velocity V by an electron of energy E into the angular range $d\psi, d\chi$ from the angles ψ_0, χ_0 with excitation of the ith channel. All recoil angles are assumed to be small and we therefore write

$$d\psi_0 = dz_0'/(L + y_0), \quad d\chi_0 = dx_0'/(L + y_0), \; \psi - \psi_0 = (z - z_0')/(L + y_0), \; \chi - \chi_0 = (x - x_0')/(L + y_0),$$

where L is the distance from the detector plane to the center of the interaction region. The total number of atoms ΔN_S scattered into $\Delta x \Delta z$ at x, z is

$$\Delta N_s = \Delta x\, \Delta z \sum_i \int J_e(x_0, y_0, z_0; E) dE dx_0 dy_0 M_a[x_0, y_0, z_0; (z_0' - z_0)/(L + y_0), (x_0' - x_0)/(L + y_0), V]$$
$$\times dz_0' dx_0' (L + y_0)^{-2} dV \sigma_i' [(z - z_0')/(L + y_0), (x - x_0')/(L + y_0); E, V]\, dz_0/(L + y_0)^2 \; . \quad \text{(B-2)}$$

Now the number of unscattered atoms reaching $dx_0' dz_0'$ per second with speeds in dV is

$$J_a(x_0', z_0'; V) dx_0' dz_0' = \left\{ \int\!\!\int M_a\left(x_0, y_0, z_0; \frac{z_0' - z_0}{L + y_0}, \frac{x_0' - x_0}{L + y_0}, V\right) \frac{dx_0 dz_0}{(L + y_0)^2}\, V dV \right\} dx_0' dz_0' \; . \quad \text{(B-3)}$$

If J_e is not a function of x_0, z_0, Eq. (B-2) can be integrated over x_0, z_0 to obtain

$$\Delta N_s = \Delta x\, \Delta z \sum_i \int J_e(y_0; E) dy_0 dE\, J_a(x_0', z_0'; V) dx_0' dz_0' (L + y_0)^{-2} \sigma_i' dV/V \; . \quad \text{(B-4)}$$

The total electron current intersecting the atom beam in the energy range dE is

$$I_e(E) dE = \int J_e(x_0, y_0; E)\, dx_0 dy_0 dE, \quad \text{(B-5)}$$

and if J_e is not a function of x_0,

$$I_e(E) dE = H \int J_e(y_0; E) dy_0 dE \; , \quad \text{(B-6)}$$

where H is the height of the scattering region. If, finally, $y_0 \ll L$ and since J_a is not a function of y_0, we perform the y_0 integration to obtain

$$\Delta N_s = (\Delta x \Delta z / H L^2) \sum_i \int I_e(E) dE\, J_a(x_0', z_0'; V) dx_0' dz_0' \sigma_i' dV/V \; . \quad \text{(B-7)}$$

A detector of negligibly small width w and non-negligible height h will therefore receive the signal

$$\Delta N_s = (w/HL^2) \sum_i \int_{-\frac{1}{2}h}^{+\frac{1}{2}h} dx \int I_e(E) dE\, J_a(x_0', z_0'; V) dx_0' dz_0' \sigma_i' dV/V \; . \quad \text{(B-8)}$$

The cross section as a function of the atom scattering angles $\sigma'(\psi', x')$ is related to the cross section as a function of the electron polar and azimuthal scattering angles θ, φ by

$$\sigma(\theta) \sin\theta d\theta d\varphi = \sigma'(\psi', \chi') d\psi' d\chi', \quad \text{where } \psi' = \psi - \psi_0, \;\; \chi' = \chi - \chi_0 \; . \quad \text{(B-9)}$$

The atom and electron scattering angles are related by

$$\psi_i' = \alpha - \beta_i \cos\theta \quad \text{(B-10)}$$

$$\chi_i' = \beta_i \sin\theta \sin\varphi \; , \quad \text{(B-11)}$$

where $\alpha = mv/MV$, and $\beta_i = mv_i'/MV$.

It has been assumed here that the momentum of the electron is small compared to that of the atom, and that the initial angle made by the atom velocity with the y axis is sufficiently small so that we may consider the angle of intersection of the atom and electron as 90°. Then

$$\sigma_i'(\psi', \chi') = \sigma_i'(\theta)/\beta_i^2 \sin\theta\cos\varphi \quad . \tag{B-12}$$

The angle θ is held fixed in performing the integration in Eq. (B-8), since $z - z_0'$ is fixed. This gives

$$\Delta N_s = (2w/HL^2)\sum_i \int_{\varphi_1}^{\varphi_2} d\varphi\, \sigma_i(\theta) I_e(E) J_a(x_0', z_0'; V) dE dx_0' dz_0' dV/\beta_i V \, , \tag{B-13}$$

where the limits of integration φ_1, φ_2 are given by

$$\varphi_{1,2} = \mp \sin^{-1}\left(\frac{\frac{1}{2}h \mp x_0}{L\beta_i \sin\theta}\right), \qquad \tfrac{1}{2}h + x_0 \leq L\beta_i \sin\theta, \tag{B-14}$$

$$\varphi_{1,2} = \mp\pi/2, \qquad \tfrac{1}{2}h - x_0 \geq L\beta_i \sin\theta \quad . \tag{B-15}$$

We define $\gamma_i(\theta, V, E) = (\varphi_2 - \varphi_1)/\pi$, called the "azimuthal form factor" (AFF), which is the fraction of the total scattered atom current collected by a detector of height h for a given polar angle θ. Note that $\gamma_i = 1$ if $h/2 - x_0 \geq L\beta_i \sin\theta$, i.e., that for this case the detector is sufficiently high so that all atoms scattered into θ are collected.

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