The potentials of a vortex are large and we have not employed the Born approximation as is usually done for scattering by atomic potentials. We have also examined a model for a vortex in which the depression in $\Delta(r)$ has a square-well appearance and the superfluid velocity is limited to the confines of the well. Such a model can be solved analytically and the results may be compared with the more realistic calculations presented here. The bound-state spectrum agrees qualitatively with that of Caroli, de Gennes, and Matricon.[7,8]

Forgan and Gough have estimated a vortex cross section of 40 Å from their measurements of the anomaly in the ultrasonic attenuation near H_{c1}.[1] More recently, Sinclair and Leibowitz have observed a much larger effect in vanadium which suggests a cross section of 200 Å near T_c for the attenuation of longitudinal sound waves propogating in the direction of the external field.[27] In order to compare calculations with experiment it is necessary to obtain a transport theory of superconductors in the presence of vortices. We are carrying out such a program and hope to publish the results soon.

ACKNOWLEDGMENTS

The author wishes to thank Dr. A. L. Fetter for suggesting this problem and for encouragement and helpful discussions during the course of the work. We also wish to thank C. Caroli for directing us to the paper of Bohm and Aharonov during our study of the 2π rotation about a vortex line, and A. C. E. Sinclair and J. Leibowitz for communicating their experimental results to us prior to publication.

[27] A. C. E. Sinclair and J. Leibowitz Phys. Rev. **175**, 596 (1968).

PHYSICAL REVIEW VOLUME 175, NUMBER 2 10 NOVEMBER 1968

Ultrasonic Investigation of the Isolated-Vortex State near H_{c1} in Vanadium*

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(Received 6 May 1968)

Ultrasonic attenuation in the mixed state of vanadium has been investigated near H_{c1}, where the vortex density is low. The expected rise toward the normal-state value of attenuation at H_{c2} is preceded by an initial absorption decrease as the vortices penetrate. The unexpected dip is interpreted in terms of a reduction in the quasiparticle mean free path, a result of the scattering of quasiparticles by vortices. The scattering width of a vortex was found to be $(2.4\pm0.4)\times10^{-6}$ cm for temperatures near T_c, in good agreement with the theoretical determination 2.1×10^{-6} cm. The apparent width decreases at lower temperatures. Similar measurements have been made in niobium. The scattering width is found to be considerably greater in vanadium, and its temperature dependence is in the opposite direction. A further feature of the present results observed even at rather low fields was the faster-than-linear dependence of the reciprocal attenuation on the internal field.

THE attenuation of ultrasonic waves in a pure type-II superconductor near H_{c1} shows an unexpected decrease as the field initially penetrates. This was first demonstrated in experiments in niobium.[1,2] We believe that the attenuation provides a sensitive test for the vortex properties, and present here results of a detailed investigation of the field dependence of the attenuation. This behavior shows a systematic trend with temperature.

The vanadium sample was the same one as that used by Radebaugh and Keesom[3] for their demonstration of the intrinsic type-II behavior of vanadium. Longitudinal 150-Mc/sec sound waves were propagated axially through the central core (diam 0.1 cm) of the cylindrical sample (length 1.9 cm, diam 0.7 cm). A magnetic field was applied along the sample axis and swept linearly in time. The voltage appearing across a short coil (length 0.3 cm), wound centrally on the sample, was integrated to give the total flux within the coil. With a correction for the flux within the air gap between coil and sample, the average magnetic field within the central region of the sample was determined from measurements in the normal and mixed states.

The magnetization through the sample was not expected to be constant since the demagnetizing coefficient for a cylindrical sample is not uniform. This consideration led us to propagate the sound wave only through the central core of the sample, so that the attenuation level was determined only by the field within this core. Thus, the effect of the nonuniformity in

* Supported in part by the Advanced Research Projects Agency.

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[1] E. M. Forgan and C. E. Gough, Phys. Letters **21**, 133 (1966).

[2] N. Tsuda and T. Suzuki, J. Phys. Chem. Solids **28**, 2487 (1967).

[3] R. Radebaugh and P. H. Keesom, Phys. Rev. **149**, 209 (1966); **149**, 217 (1966). The reader is referred to these papers for a complete description of the superconducting properties of vanadium, and of this particular sample.

the demagnetizing coefficient was minimized. There was also the possibility of radial field gradients, which arise from flux pinning. We believe that such gradients were small, from the results of hysteresis measurements on tubes and bulk specimens.[4] Thus, on the assumptions of negligible end effects and approximate field uniformity radially within the sample, the internal field was taken to be uniform and equal to the average field determined by the coil. An evaporated surface coating of copper was applied to the vanadium in order to minimize hysteretic currents and to allow flux penetration to take place more smoothly.

The attenuation and magnetization were simultaneously measured as the external field was varied, for several temperatures. For each run the sample began in the virgin state, having been raised above the critical temperature T_c and cooled in a very small magnetic field (less than 0.5 G). The field was swept sufficiently slowly for the measurements to be independent of sweep rate. Experimentally, the main limitation on accuracy was the stability of the electronic equipment, which gave jumps and drifts in the attenuation level of about 0.04 dB. In order to maximize attenuation differences, observations were made for the longest possible sound path length, consistent with a satisfactory signal-to-noise ratio and the limitation of input

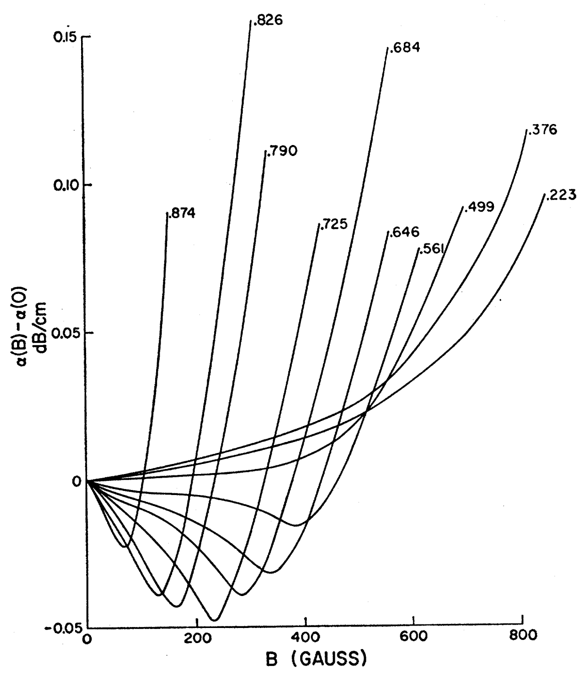

FIG. 1. Change in attenuation from the zero-field value, plotted against the *internal* field. The reduced temperature T/T_c is shown for each curve.

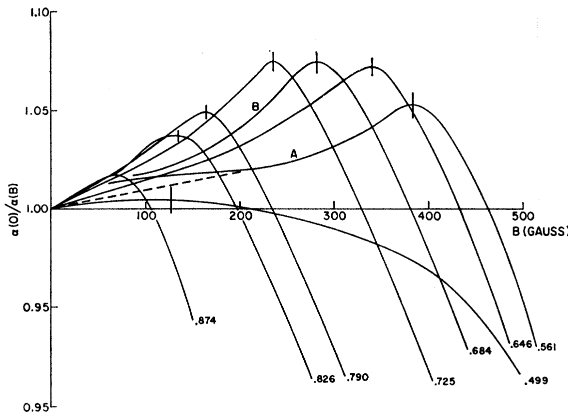

FIG. 2. Field dependence of the reciprocal of the normalized attenuation, for several reduced temperatures.

power. This power was limited to a level which produced no heating of the sample (by the ultrasonic energy) and no receiver saturation due to the leakage pulse between transmitter and receiver. At the highest temperature used here, a path of 13.2 cm was used. This was increased to 28.2 cm at lower temperatures.

The attenuation data of several runs at each temperature were averaged and are shown in Fig. 1. The difference between the attenuation $\alpha(B)$ in the internal field B and zero-field attenuation $\alpha(0)$ is plotted against the internal field. The magnitude of $\alpha(0)$ and its temperature dependence were approximately that expected from the Bardeen-Cooper-Schrieffer prediction, given the measured attenuation level of 2.84 dB/cm in the normal state.

The initial decrease in attenuation on first penetration of vortices was followed by the expected increase toward the normal-state value as the external field was increased toward H_{c2}. The initial decrease can be understood in terms of the model proposed by Forgan and Gough.[1] In this model the spectrum of thermally excited quasiparticles is treated as unchanged by the presence of vortices. The vortices simply scatter the quasiparticles, and the strength of the interaction between a quasiparticle and a vortex is described by the scattering width a of the vortex. The attenuation is thus the usual superconducting attenuation, except that the quasiparticles now have a shorter mean free path. A microscopic theory to describe the quasiparticle scattering has recently been given by Cleary.[5]

On the assumption of independent impurity and vortex scattering mechanisms, the electron mean free path l is given by

$$l^{-1}=(l_{\text{imp}})^{-1}+(l_{\text{vort}})^{-1}, \tag{1}$$

where l_{imp} and l_{vort} are the mean free paths determined

[4] See, e.g., D. G. Schweitzer, M. Garber, and B. Bertman, Phys. Rev. 159, 348 (1967).

[5] R. M. Cleary (to be published).

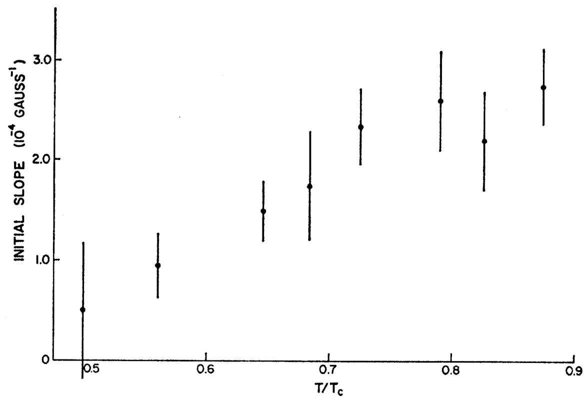

FIG. 3. Temperature dependence of the initial slopes of the curves of Fig. 2.

by impurities and vortices, respectively. With a vortex surface density N across the plane perpendicular to the field we have, for low vortex density,

$$(l_{\text{vort}})^{-1}=Na. \tag{2}$$

The ultrasonic attenuation under conditions such that $ql\ll 1$, where q is the sound wave number, is proportional to l.[6] For the present measurements[3] $ql_{\text{imp}}=0.03$, which is well within the required condition. Since $N=B/\phi_0$, where ϕ_0 is the flux quantum, we have the Forgan-Gough expression

$$\alpha(0)/\alpha(B)=1+(Bal_{\text{imp}}/\phi_0). \tag{3}$$

Figure 2 shows the experimental data plotted in accordance with Eq. (3). The error bars represent the jumps and drifts in the attenuation level, and were obtained by normalizing the possible error of 0.04 dB by the zero-field attenuation appropriate to each temperature. Curves A and B have an apparent shoulder at small fields. The shoulder heights are comparable with the possible errors, however, so that it is not possible to consider them significant. The extrapolation of the higher-field data to zero field is indicated for curve A by the dashed line. With this extrapolation, the initial slope of each curve shows a systematic increase with increasing temperature.

Although Eq. (3) predicts linearity with B, $\alpha(0)/\alpha(B)$ rises faster than linearly at rather low fields. For a hexagonal vortex lattice, the vortex spacing exceeds l_{imp} (2.45×10^{-5} cm) at fields less than 400 G. Thus the faster-than-linear rise occurs at fields such that the vortex spacing is large compared with the characteristic lengths of the system l_{imp} and the coherence length ξ_0 (4.5×10^{-6} cm). It would appear, then, that the assumption of isolated vortices breaks down even for widely spaced vortices.

If we assume that the quasiparticle scattering model is correct at very low vortex density, then the initial slope of the curves will determine the scattering width a. The initial slope is given in Fig. 3 as a function of temperature. Comparison with the theoretical prediction of Cleary[5] is possible near T_c, where the theory should be valid. The theory was developed to describe the properties of isolated vortices in a type-II superconductor with $\kappa\simeq0.707$, and so is applicable to both vanadium and niobium. The calculation of the quasiparticle-vortex interaction for vanadium gave a to be 2.1×10^{-6} cm. The experimental value, for $T=0.9T_c$, is $(2.4\pm0.4)\times10^{-6}$ cm. As yet there is no theoretical estimate for the temperature dependence of a. Experimentally the *apparent* temperature dependence is large, with a increasing monotonically from zero, at about $0.45T_c$, as the temperature increases.

It should be remembered, however, that the model which we are using for the field dependence of the attenuation is too simple. We should also include the effect of the spatially varying quasiparticle spectrum,[7] of the contributions from excitations bound in the neighborhood of vortex cores,[8] and of the modification to these states by the periodic nature of the vortex lattice.[9] These effects give rise to a "background" attenuation, in terms of the present model, and lead to the rise in attenuation at high fields. The low-field dependence of this background attenuation will determine, in part, the initial slopes of the curves in Fig. 2, and hence the temperature dependence shown in Fig. 3. Since the attenuation $\alpha(0)$ decreases to zero with decreasing temperature, we may expect the background attenuation to become increasingly important as the temperature is lowered.

Finally, it is interesting to compare the present results with those from the earlier work in niobium. The attenuation dip was present in niobium, as noted previously, but the scattering width was estimated to be 1.5×10^{-7} cm.[10] This is much smaller than the value found in vanadium, and also much smaller than the theoretical prediction. The scattering width above $0.6T_c$ was found to *decrease* with increasing temperature, again in contrast with the present results. The qualitative and quantitative differences are remarkable in view of the otherwise very similar superconducting behavior of the two metals.

We are grateful to Dr. G. A. Alers of the Ford Scientific Laboratory for the loan of the vanadium sample.

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[10] From Ref. 1, with the correction given in Ref. 5. A very rough value for the scattering width can be derived from Ref. 2 and appears to be of the order of 1.7×10^{-6} cm, approximately temperature-independent.