anisotropic, although the gap in any direction was within 10% of the BCS value of $3.52kT_c$. The values of the energy gaps from this work are somewhat different from those quoted from other ultrasonics measurements in thallium. However, it is believed that the present results are more accurate.

α_s/α_n was found to be frequency-dependent for propagation along the [10$\bar{1}$0] direction. This behavior is similar to that observed in measurements on lead, where it was attributed to a change in the phonon-limited electron mean free path between the normal and superconducting states.

ACKNOWLEDGMENTS

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Ultrasonic Attenuation in the Normal State of High-Purity Thallium*

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The normal-state electronic attenuation of longitudinal ultrasonic waves (10 to 130 MHz) has been studied in high-purity thallium. The attenuation is in general agreement with the Pippard real-metal theory, but does not agree with the free-electron theory. The attenuation is highly anisotropic, in part because of anisotropies in the electron mean free path, the total curvature of the Fermi surface, and the product of the curvature and the Pippard deformation parameter. The electron mean free path was determined from the acoustic measurements for $\mathbf{q}\,\|[0001]$, $\mathbf{H}\,\|[10\bar{1}0]$ and $\mathbf{q}\,\|[\bar{1}2\bar{1}0]$, $\mathbf{H}\,\|[10\bar{1}0]$. The results indicate that the anisotropy in the electron mean free path is due to impurity scattering. The phonon scattering contribution was proportional to T^{-b}, where $b=3.6\pm0.1$ for both directions, whereas $b=5$ has been reported from electrical resistivity measurements. A frequency-dependent component of the electronic attenuation was observed for $\mathbf{q}\,\|[0001]$ which could not be explained on the basis of the Pippard theories.

I. INTRODUCTION

THIS paper reports on the attenuation of longitudinal ultrasonic waves by electrons in the normal state of high-purity thallium at frequencies from 10 to 130 MHz and over the temperature range 0.5 to 4.2°K. Propagation was along the principal directions with and without an applied transverse magnetic field. The specimen used was the same as that used in the study of the superconducting-state attenuation reported in the previous paper.[1] The results of the normal-state study permitted an accurate estimate to be made of the zero-magnetic-field attenuation below the superconducting transition temperature, necessary to complete the study of the superconducting-state attenuation. In addition, we have compared the normal-state attenuation with the Pippard free-electron[2] and real-metal theories.[3] In the following discussion we will briefly review the results of the Pippard theories of most interest for this work. Further results of Pippard and others will be presented where appropriate in the text.

The amplitude attenuation of ultrasonic waves for zero applied magnetic field H due to free electrons on a spherical Fermi surface is given by

$$\alpha_n(0,\,ql)=\left[\frac{\pi^2}{6}\frac{Nmv_f}{\rho v_s}f\right]\left[\frac{2}{\pi}\frac{ql\tan^{-1}ql}{ql-\tan^{-1}ql}-\frac{6}{\pi ql}\right] \quad (1)$$

$$=\alpha_n(0,\,\infty)P(ql), \quad (2)$$

where we have written the attenuation in the normal state as an explicit function of ql and H as $\alpha_n(H,\,ql)$, and q is the wave number of the sound wave and l is the electron mean free path. The terms in Eq. (2) correspond, respectively, to the two bracketed terms in Eq. (1). Here N is the density of free electrons, m is the electron mass, v_f is the Fermi velocity, ρ is the density of the metal, v_s is the velocity of sound, and f is the frequency of the ultrasonic wave. For $ql\ll1$, the attenuation is due to electrons from over the entire Fermi surface; whereas for $ql\gg1$, the effective electrons are largely those lying in a narrow effective zone [of angular width $\phi\sim(ql)^{-1}$], defined by the equation $v_f\cdot\mathbf{q}\simeq0$. Thus for the free-electron sphere, the only anisotropy in the attenuation for any value of ql is that due to the sound velocity.

Pippard has shown that the attenuation for real

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[1] H. J. Willard, Jr., R. W. Shaw, and G. L. Salinger, preceding paper, Phys. Rev. 175, 362 (1968).

[2] A. B. Pippard, Phil. Mag. 46, 1104 (1955).

[3] A. B. Pippard, Proc. Roy. Soc. (London) A257, 165 (1960).

metals in the limit $ql \gg 1$ is independent of l and is given by

$$\lim_{ql\to\infty} \alpha_n(0, ql) = \frac{\hbar q}{4\pi^2 \rho v_s} \oint RK_x^2 d\psi, \tag{3}$$

where K_x is the component of the deformation parameter $\mathfrak{D}$ on the effective zone in the direction of propagation, R is the product of the principal radii of curvature on the effective zone, and the integration is around the effective zone. Therefore, anisotropy in the case of real metal may be due to l, $\mathfrak{D}$, R, and v_s.

In the presence of a large, transverse magnetic field Pippard found that, for both free-electron and real metals,

$$\alpha_n(H, ql) \propto H^{-2} \tag{4}$$

for all ql; whereas for lower H and $ql \simeq 1$, the attenuation is periodic in H^{-1}, exhibiting magnetoacoustic oscillations. In the limit of infinite transverse magnetic-field strength, the attenuation is proportional to $q^2 l$. For the free-electron case, $a_n(\infty, ql)$ is explicitly given by

$$\alpha_n(\infty, ql) = (2/5\pi)\alpha_n(0, \infty)ql. \tag{5}$$

The Fermi surface of thallium is known to be nearly spherical in the extended-zone scheme.[4–7] However, in the reduced-zone scheme, it extends over six Brillouin zones, appears to partially fill four zones, and is extremely anisotropic. Weil and Lawson[8] have shown that the frequency and velocity dependences of shear and longitudinal waves agree with the Pippard theories for $ql \ll 1$, and that the attenuation in this ql region is isotropic if account is taken of the difference in sound velocities. This implied good agreement with real-metal theory, but did not determine whether or not the attenuation agreed with the free-electron theory. We have found for $ql > 1$, in general, good agreement with the real-metal theory, but very poor agreement with the free-electron theory. Furthermore, we have observed a component of the electronic attenuation not accountable with the Pippard theory.

II. EXPERIMENTAL DETAILS

The thallium specimen, approximately 1 cm on a side, was spark cut from a single-crystal portion of an ingot prepared by the strain-anneal technique[9] from material of nominally 99.999% starting purity supplied by United Mineral and Chemical Co. The ultrasonic results indicate that high purity and crystalline perfection were maintained in the specimen preparation. Crystal orientations were accurate to $\pm 2°$. Opposite faces were maintained parallel to within 0.0001 in. and reasonably free of oxide by occasional replaning and light etching. Optically polished 10-MHz transducers were bonded to the specimen with No-naq stopcock grease or Dow Corning high-vacuum grease. The latter gave poorer coupling but was sometimes necessary when high humidity caused trouble with the water soluble No-naq.

The electronic system was basically that described by Love and Shaw.[10] Frequency measurements were made using a calibrated oscilloscope with a maximum uncertainty of $\pm 3\%$. The uncertainty in the normal-state attenuation was estimated at $\pm 2\%$.

Most of the normal-state measurements considered here were taken in a He^4 cryostat capable of a minimum temperature of 1.2°K. In this system the specimen and mount resided within a tube immersed in the liquid helium. The tube was normally filled with He gas at a pressure of several hundred microns of Hg during the measurements to minimize temperature gradients. Superconducting-state results down to 0.5°K, taken in a He^3 cryostat and reported in detail in the accompanying paper, are used here to establish the level of nonelectronic attenuation in the various directions.

Temperatures between 1.2 and 4.2°K were measured to an estimated accuracy of ± 0.010°K using a Speer carbon resistor previously calibrated against He^4 vapor pressure. Magnetoresistance was observed in the resistor, the change in resistance being approximately proportional to field strength. At 1.2°K the change of resistance at 9400 Oe corresponded to a temperature change of -0.035°K. Corrections for this effect were made routinely. The magnetic field was provided by a Harvey-Wells 12-in. magnet and was known to 1%.

III. RESULTS

Figures 1, 2, and 3 are representative of data taken in each crystalline direction for the attenuation of ultrasonic waves at 30 MHz as a function of temperature with and without an applied magnetic field. The

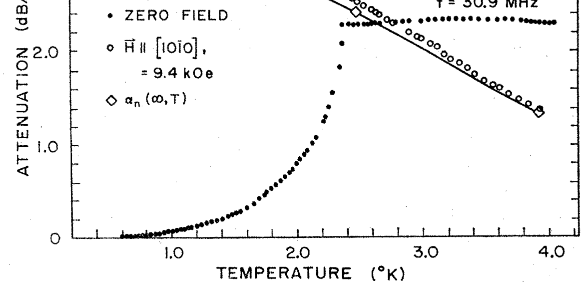

FIG. 1. Zero-field attenuation and transverse-field attenuation at 9.4 kOe versus temperature for **q** || [0001], **H** || [10$\bar{1}$0], f= 30.9 MHz. Infinite-field attenuation points and the infinite-field curve are also shown.

[4] P. Soven, Phys. Rev. **137**, A1706 (1965); **137**, A1717 (1965).
[5] J. A. Rayne, Phys. Rev. **131**, 653 (1963).
[6] Y. Eckstein, J. B. Ketterson, and M. G. Priestly, Phys. Rev. **148**, 586 (1966).
[7] W. A. Harrison, Phys. Rev. **118**, 1190 (1960).
[8] R. B. Weil and A. W. Lawson, Phys. Rev. **141**, 452 (1966).
[9] M. Tannenbaum, in *Methods of Experimental Physics*, edited by K. Lark-Horowitz and Vivian A. Johnson (Academic Press Inc., New York, 1959), Vol. 6, p. 86.
[10] R. E. Love and R. W. Shaw, Rev. Mod. Phys. **36**, 260 (1964).

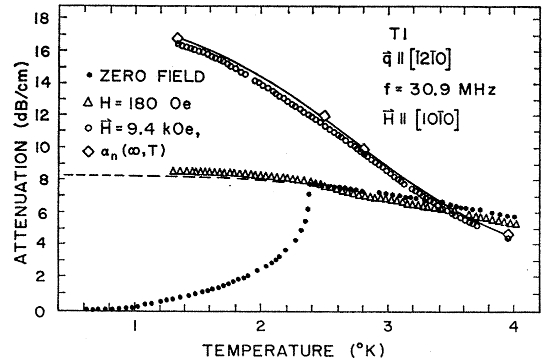

FIG. 2. Zero-field attenuation and transverse-field attenuation (H=180 Oe, 9.4 kOe) versus temperature for **q** ∥ [1̄21̄0], **H** ∥ [101̄0], f=30.9 MHz. Also shown are the infinite-field points, infinite-field curve, and the estimated zero-field curve for $T \leq T_c$ (dashed line).

dashed curves are the estimated zero-field curves for $T<T_c$, used in evaluating the superconducting-state attenuation presented in the accompanying paper. These figures also illustrate the unusual temperature dependences of the zero-field attenuation in the [0001] direction and the infinite-magnetic-field attenuation in the [101̄0] direction. These anomalies will be discussed below.

The frequency dependence of the zero-field attenuation at 2.38°K (T_c) is shown in Fig. 4. The attenuation exhibits linearity with frequency at the higher frequencies, which implies $ql \gg 1$, in contrast to the f^2 dependence observed by Weil and Lawson[8] for relatively impure thallium. The data for **q** ∥ [0001], although of the same trend, do show deviation from that observed in the other two directions. Table I presents the limiting slopes of these curves at high frequencies, along with the calculated free-electron values and the sound velocities. The anisotropy in the measured values are seen to be too large to be resolved through v_s. According to Eq. (3), this anisotropy is due to anisotropy in RK_x^2.

The zero-field attenuation in the [0001] direction also showed anomalous behavior with temperature,

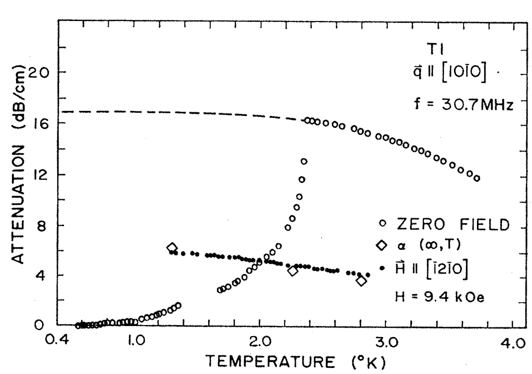

FIG. 3. Zero-field attentuation and transverse-field (H=9.4 kOe) attentuation for **q** ∥ [101̄0], **H** ∥ [1̄21̄0], f=30.7 MHz. Infinite-field points and the estimated zero-field attenuation for $T \leq T_c$ (dashed line) are also shown.

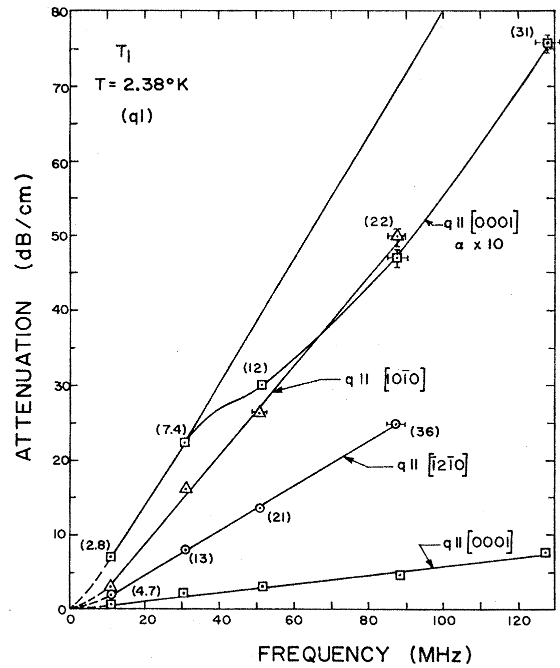

FIG. 4. Attenuation at 2.38°K frequency with ql values (bracketed) determined from the high-field technique at 30 MHz.

manifested by weak maxima and minima in the attenuation for $T<3.5$°K, observed for all frequencies except 10 MHz. The amplitude of the undulations was less than 10% of the normal-state attenuation at T_c and was always reproducible. There were no amplitude-dependent effects observed for this specimen.[1] In addition, an amplitude-independent, temperature-dependent, background attenuation was not likely the cause of the undulations because (a) they were not observed for any other direction and (b) the temperature dependence of the phonon-limited electron mean free path (to be discussed below) was the same for propagation in both the [0001] and [1̄21̄0] directions. Furthermore, two different bonding agents were used, so that it was not likely an effect due to bonds.

The mean-free-path results, to be presented below, are made use of here in order that a detailed comparison with the Pippard free-electron theory can be made. In Figs. 5 and 6, $\alpha_n(0, ql)/f$ is plotted against ql for the [1̄21̄0] and [0001] directions, respectively. ql could

TABLE I. Sound velocities at 4.2°K, experimental and calculated free-electron values of the slopes of the longitudinal wave attenuation versus frequency curves for $ql \gg 1$.

	v_s [a] (10^5 cm/sec)	$\alpha_n(0, \infty)/f$ (dB/cm)/MHz	$d\alpha/df\,\vert_{ql\gg1}$ (dB/cm/MHz)
		Free electron	Experiment
[0001]	2.25	0.39	0.07±0.02
[1̄21̄0]	1.94	0.52	0.60±0.05
[101̄0]	1.94	0.52	0.30±0.02

[a] Reference 8.

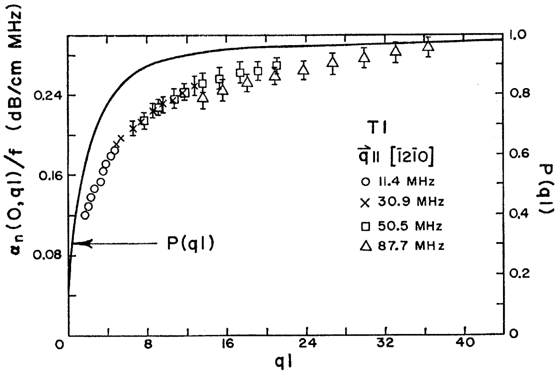

FIG. 5. $\alpha_n(0, q)/f$ versus ql for $\mathbf{q} \parallel [\bar{1}2\bar{1}0]$. $P(ql)$ versus ql is also shown, scaled to the value of the slope $d\alpha/df$ for $ql \gg 1$, given in Table I.

not be determined for propagation along the $[10\bar{1}0]$ direction. The Pippard function $P(ql)$, defined in Eqs. (1) and (2), is also shown, normalized to the measured value of

$$(d\alpha/df)_{ql\gg1}=\alpha_n(0,\ \infty)/f$$

presented in Table I. It is easily seen that the attenuation does not agree with the free-electron theory. Attempts to improve the fit of the data to the Pippard function by multiplying ql by a constant were not significantly successful.

The magnetic field was aligned along the $[10\bar{1}0]$ direction for $\mathbf{q} \parallel [0001]$ and $[\bar{1}2\bar{1}0]$ and along the $[\bar{1}2\bar{1}0]$ direction for $\mathbf{q} \parallel [10\bar{1}0]$. Magnetoacoustic oscillations were observed in all three cases. A rough estimate of the electron mean free path at 1.30°K was made from the fact that oscillation should be appreciable only when $\omega_c\tau \geq 1$. For $H=1000$ Oe, $v_f=1.7\times10^8$ cm/sec and m taken to be the free-electron $\omega_c\tau=1$ when $l=0.01$ cm, in good agreement with l determined from the high-field measurements. Free-electron theory predicts that the oscillation should be periodic in H^{-1} and should diminish in amplitude as H decreases.[11] This behavior was observed for propagation

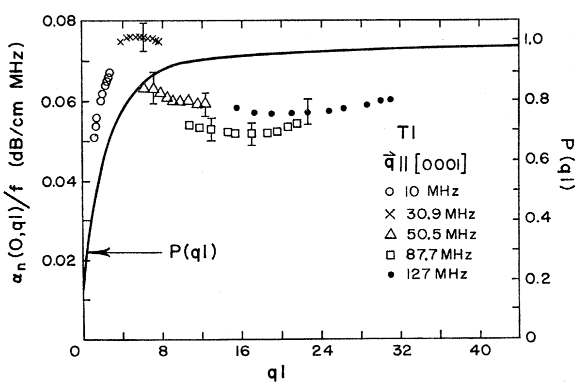

FIG. 6. $\alpha_n(0, ql)/f$ versus ql for $\mathbf{q} \parallel [0001]$. $P(ql)$ versus ql is also shown, scaled to the value of the slope $d\alpha/df$ for $ql \gg 1$ given in Table I.

[11] T. Kjeldaas, Jr., and T. Holstein, Phys. Rev. Letters **2**, 260 (1964).

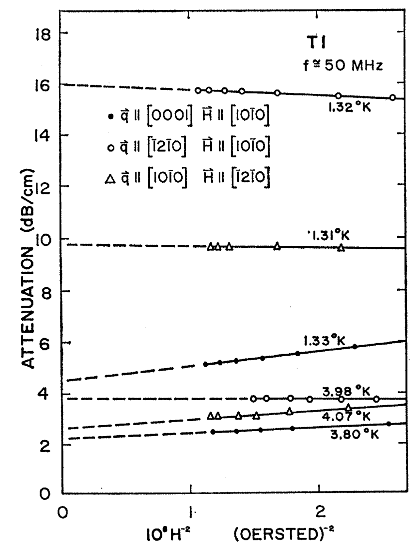

FIG. 7. Typical attenuation versus H^{-2} curves.

along the $[0001]$ and $[\bar{1}2\bar{1}0]$ directions; however, the attenuation continued to increase with H^{-1} for $\mathbf{q} \parallel [10\bar{1}0]$.

In Fig. 7 are shown representative samples of the 50-MHz attenuation at high fields as a function of H^{-2}. The H^{-2} dependence was observed at all frequencies and temperatures for each direction. Such plots were extrapolated to $H^{-2}=0$ to obtain the attenuation at infinite magnetic field strength.

As previously discussed, the infinite-field attenuation is proportional to the electron mean free path and to the square of the frequency. It appears from Fig. 8 that the f^2 dependence is reasonably well followed for $\mathbf{q} \parallel [\bar{1}2\bar{1}0]$ and $[10\bar{1}0]$. The disagreement at 30 MHz for $\mathbf{q} \parallel [10\bar{1}0]$ may have been due to misalignment of the magnetic field. Measurements at 50 and 90 MHz were made during the same run with the same orientation of the magnetic field to within $\pm\frac{1}{4}°$. The 30-MHz data were from a different run and the alignment of the field was only to within $\pm3°$.

It follows from the Pippard theory that the infinite-

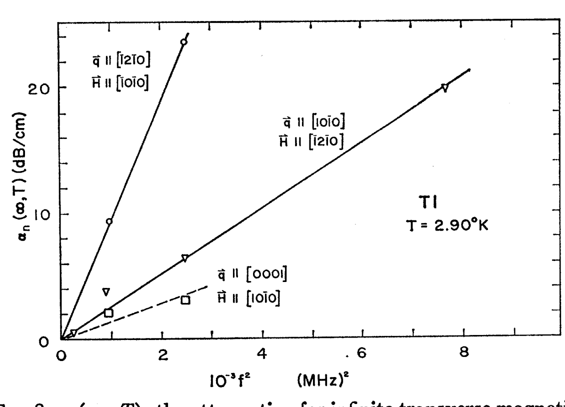

FIG. 8. $\alpha_n(\infty, T)$, the attenuation for infinite transverse magnetic-field strength versus frequency squared at $T=2.90°$K.

TABLE II. Comparison of normal-state electronic attenuation in thallium with Pippard theories for $ql>1$.

	$\alpha_n(0,ql)/P(ql)$ =const	$[\alpha_n(0,ql)/f]_{ql\gg1}$ =const	$\alpha_n(\infty,ql)/f^2$ =const	$\alpha_n(H,ql)/H^{-2}$ =const	$f^{-1}[l(T)/l(T_0)]$ =const
q ‖ [0001] **H** ‖ [10$\bar{1}$0]	No	No	No	Yes	Yes
q ‖ [$\bar{1}$2$\bar{1}$0] **H** ‖ [10$\bar{1}$0]	No	Yes	Yes	Yes	Yes
q ‖ [10$\bar{1}$0] **H** ‖ [$\bar{1}$2$\bar{1}$0]	...	Yes	Yes	Yes	No

magnetic-field attenuation equals the zero-field attenuation when $ql=6.8$.[12] Since $\alpha_n(\infty, ql)$ is proportional to ql (at constant frequency), it is expected that the infinite-field attenuation can greatly exceed the zero-field attenuation at low temperatures in a high-purity sample (i.e., $ql\gg6.8$). This was observed for **q** ‖ [0001] and [$\bar{1}$2$\bar{1}$0] (Figs. 1 and 2); however, the high-field attenuation was very low for **q** ‖ [10$\bar{1}$0] (Fig. 3). Since magnetoacoustic oscillation was observed in this direction, it is unlikely that the low attenuation is due to low ql. Figures 1, 2, and 3 also show curves of the attenuation at high fields ($H=9000$ Oe) as a function of temperature, which were of great value in interpolating between the infinite-field points.

ql (at constant q), as a function of temperature, was determined for **q** ‖ [0001] and [$\bar{1}$2$\bar{1}$0] using the relation

$$ql(T)=6.8[\alpha_n(\infty, T)/\alpha_n(\infty, T_1)], \qquad (6)$$

where T_1 is the temperature at which $\alpha_n(\infty, ql)=\alpha_n(0, ql)$. Figure 9 shows l as a function of temperature

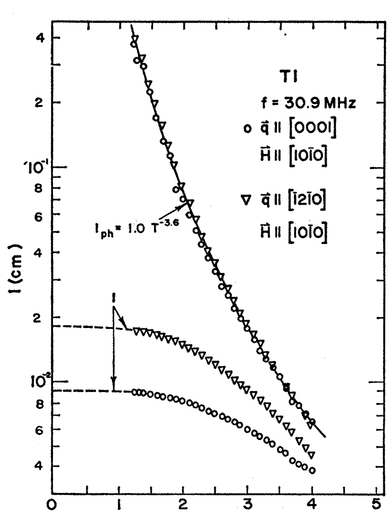

FIG. 9. l and l_{phonon} versus T for **q** ‖ [0001], **H** ‖ [10$\bar{1}$0] and **q** ‖ [$\bar{1}$2$\bar{1}$0], **H** ‖ [10$\bar{1}$0] at $f=30.9$ MHz.

[12] B. C. Deaton and J. D. Gavenda, Phys. Rev. **129**, 1990 (1963).

for both directions from data taken at 30 MHz. A separation of mean-free-path limits due to phonon and impurity scattering can be accomplished by making the standard assumptions[13] that (a) all scattering in the limit $T\to0$ is because of impurities, (b) this mechanism is independent of temperature in the range studied, and (c) the two scattering mechanisms are independent, i.e., $l^{-1}=l_i^{-1}+l_{\text{ph}}^{-1}$. Here l_i is the impurity-scattering-limited mean free path, and l_{ph} is the phonon-scattering-limited mean free path. Extrapolating these curves to $T=0$°K gives $l_i=1.1\times10^{-2}$ cm for **q** ‖ [0001] and $l_i=2.3\times10^{-2}$ cm for **q** ‖ [$\bar{1}$2$\bar{1}$0]. Log-log plots of $(l_{\text{ph}})^{-1}=(l)^{-1}-(l_i)^{-1}$ versus T showed a temperature dependence of T^{-b}, where $b=3.6\pm0.1$ for both directions. The resulting curve (labeled l_{ph}), along with the data, are shown in Fig. 9 where it is seen that

$$l_{\text{ph}}=1.0T^{-3.6\pm0.1}. \qquad (7)$$

The mean free paths were also determined from measurements at 50 MHz. The temperature dependence of l (and therefore l_{ph}) were found to be the same; however, l (50 MHz) $=0.95l$ (30 MHz) for **q** ‖ [$\bar{1}$2$\bar{1}$0], and l (50 MHz) $=0.67l$ (30 MHz) for **q** ‖ [0001].

Although l could not be determined from the measurements for **q** ‖ [1010], **H** ‖ [$\bar{1}$2$\bar{1}$0], the theory predicts that its temperature dependence should be discernible. However, it was found that $\alpha_n(\infty, T)/\alpha_n(\infty, T_0)$, where T_0 is a reference temperature, was different at 50 and 90 MHz. Furthermore, the T dependence of l_{ph} determined from these data did not follow a simple power law and was frequency-dependent.

IV. DISCUSSION OF RESULTS

A summary of the comparison of the experimental results with Pippard theory of the normal-state attenuation is presented in Table II. The first column refers to a comparison of the ql dependence of the zero-field attenuation with free-electron theory. The lack of agreement, as well as the large anisotropy in the attenuation, demonstrates that the free-electron theory is not adequate to explain the electronic acoustic properties of

[13] C. Kittel, *Solid State Physics* (John Wiley & Sons, Inc., New York, 1962).

thallium. Table II also shows that the attenuation for $\mathbf{q} \parallel [\bar{1}2\bar{1}0]$, $\mathbf{H} \parallel [10\bar{1}0]$ is in very good agreement with the real-metal theory, but that significant deviations from the theory exist for propagation along the other two directions.

For $\mathbf{q} \parallel [\bar{1}2\bar{1}0]$, $\mathbf{H} \parallel [10\bar{1}0]$, the frequency dependence of the zero-field and infinite-magnetic-field attenuation and the H^{-2} dependence of the high-field attenuation agreed with the real-metal theory. The last column of Table II shows that the temperature dependence of the electron mean free path did not vary with frequency—an independent check on the measurements. The magnitude of ql was based on the free-electron theory and therefore must be considered only as an estimate until theoretical expressions for the ql dependence of the zero- and infinite-field attenuations, based on the actual Fermi surface of thallium, are derived. The fact that the magnitude of l was the same from both 30- and 50-MHz measurements does suggest, however, that the actual value of ql for the condition $\alpha_n(0, ql) = \alpha_n(\infty, ql)$ will not be very different from the free-electron value of 6.8.

The frequency dependence of $\alpha_n(0, ql)$ and $\alpha_n(\infty, ql)$ and the H^{-2} dependence of the high-field attenuation were in reasonable agreement with theory for $\mathbf{q} \parallel [10\bar{1}0]$, $\mathbf{H} \parallel [\bar{1}2\bar{1}0]$; however, $\alpha_n(\infty, ql)$ was anomalously low. Since the attenuation saturated, it would appear that the anomalous behavior is not due to open orbits.[3] Pippard[3] has noted that if the number of electronlike and holelike orbits are equal for a given directionof $\mathbf{H}$ and $\mathbf{q}$, the high-field attenuation still approaches saturation as H^{-2}; and $\alpha_n(\infty, ql)$, although still proportional to q^2l, may be considerably lower than if they are unequal. The attenuation in this case is also a strong function of the cross-sectional area of the Fermi surface cut by a plane k_z, where k_z is the component of the Fermi-surface wave vector in the direction of $\mathbf{H}$. We have found the temperature dependence of the infinite-field attenuation to be different at each frequency measured. This may have been because of a high sensitivity of cross-sectional area to the direction of the applied field. Examination of the Fermi surface shows that this sensitivity[4] is indeed quite likely. More detailed measurements are required before a definite conclusion can be reached.

The attenuation for $\mathbf{q} \parallel [0001]$, $\mathbf{H} \parallel [10\bar{1}0]$ exhibited an anomalous frequency dependence in both $\alpha_n(0, ql)$ and $\alpha_n(\infty, ql)$, and an anomalous temperature dependence of the zero-field attenuation. However, the temperature dependence of l did not vary with frequency and l_{phonon} was found to agree with that of $\mathbf{q} \parallel [\bar{1}2\bar{1}0]$, $\mathbf{H} \parallel [10\bar{1}0]$ at 30 MHz. For these reasons and others previously presented, we believe that these anomalies are not due to a frequency- and temperature-dependent background attenuation, but a component of the electronic attenuation not accountable for by the Pippard theory. We have no explanation for these anomalies, but note that the Debye temperature of thallium has an unusual temperature dependence, exhibiting a maximum at about 3°K and dropping by about 8% to a constant value at 0.7°K.[14]

The electron mean free path was found to be anisotropic (Fig. 9). It seems most likely that anisotropy is due to impurity scattering in view of the agreement of l_{ph} for the measurements at 30 MHz. The temperature dependence of l_{ph} was given by T^{-b}, where $b = 3.6 \pm 0.1$. However, a T^{-5} dependence has been found for l_{ph} determined from electrical resistivity measurements.[15]

Deaton[16] has found a similar difference in the temperature dependence of l_{ph} for acoustic and electrical resistivity measurements for the hcp metals Zn and Cd which he attributed to the temperature dependence of the Debye temperature. However, the Debye temperature variation should affect both the acoustic and the conductivity determined mean free paths and on this basis there should be no difference between the two.[17] Pippard has noted that for multiband metals a difference may exist. Steinberg[18] has made estimates of the difference to be expected for shear waves in the region $ql \ll 1$, but no theory has been advanced for longitudinal waves and $ql \gg 1$.

V. CONCLUSIONS

The longitudinal-wave attenuation due to electrons in thallium is in general agreement with real-metal theory of Pippard, but does not agree with the free-electron theory. The attenuation is highly anisotropic, in part due to anisotropies in the electron mean free path, the local curvature of the Fermi surface, and the product of the curvature and the Pippard deformation parameter squared.

The electron mean free path determined from the acoustic measurements is anisotropic and is different from that obtained from electrical resistivity measurements. The results indicate that the anisotropy is due to impurity scattering.

A frequency-dependent component of the electronic attenuation was observed for propagation along the hexad axis which could not be explained on the basis of the Pippard theories.

ACKNOWLEDGMENTS

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[14] B. J. C. van der Hoeven, Jr., and P. H. Keeson, Phys. Rev. **135**, A631 (1964).

[15] N. V. Zavaritskii, Zh. Eksperim. i Teor. Fiz. **39**, 1571 (1960) [English transl.: Soviet Phys. —JETP **12**, 1093 (1961)].

[16] B. C. Deaton, Phys. Rev. **140**, A2051 (1965).

[17] J. M. Ziman, *Theory of Solids* (Cambridge University Press, Cambridge, 1964).

[18] M. S. Steinberg, Phys. Rev. **109**, 1486 (1958).