Level Density of Light Nuclei

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The level density of light nuclei ($A \lesssim 70$) is studied by analyzing statistical reaction spectra, slow-neutron resonances, and level widths at high excitation energy. Further evidence seems to indicate that, as proposed in a preceding work, the effective excitation energy of excited nuclei, appearing in the Lang–Le Couteur level-density formula, must be calculated by means of the following expression:

$$U=(E-\Delta+70/A)\ \text{MeV}.$$

E is the usual excitation energy, Δ is the pairing energy, and $70/A$ MeV is a term that shifts, in a smooth manner, the zero of the energy scale of different nuclei. In this case the Lang–Le Couteur formula, assuming $R\cong 1.5A^{1/3}$ F, $\mathcal{I}=0.7\mathcal{I}_{\text{rig}}$, $a=(0.127A)$ MeV^{-1}, gives, without further changes of its parameters, a correct estimation of the slope and absolute value of the level densities of light nuclei for energies ranging from $(1+\Delta)$ up to $\sim$20 MeV. It is also shown that the preceding choice of the parameters of Lang–Le Couteur level-density expression allows one to reproduce very well the experimental distributions of low-energy levels observed with reactions which proceed, at least partially, through the formation of a compound nucleus.

1. INTRODUCTION

VARIOUS authors have derived, on the basis of particular nuclear models, theoretical expressions for the level density $\rho(E,J)$ of atomic nuclei.[1–8]

The so called "equidistant spacing model" is the most widely used; the estimation of the single-nucleon spacing necessary to calculate the level density of the nucleus[5] is usually performed by utilizing the Fermi-gas model or the shell-model nucleon density averaged over a small energy interval around the Fermi energy. A general survey of mathematical methods and physical assumptions is given in Ref. 6.

Some discrepancies concerning the correct expression of the level density $\rho(E,J)$ appear in the literature.

For the most part two different expressions are used:

$$\rho(E,J)=\frac{\hbar^3}{12(8)^{1/2}}(2J+1)\times\exp\left[-\frac{J(J+1)}{2\sigma^2}\right]a^{1/2}\mathcal{I}^{-3/2}\frac{\exp[2(aU)^{1/2}]}{U^2},\quad(1)$$

$$\rho(E,J)=\frac{\hbar^3}{12(8)^{1/2}}(2J+1)\times\exp\left[-\frac{J(J+1)}{2\sigma^2}\right]a^{1/2}\mathcal{I}^{-3/2}\frac{\exp[2(aU)^{1/2}]}{(U+t)^2}.\quad(2)$$

$\rho(E,J)$ is the density of levels having energy E and spin J. The density of levels of a given parity π is just one-half of this quantity if one assumes an equal probability of both parities. This is usually considered to be the case for excitation energies which are not too low. U is an "effective excitation energy" simply related to the excitation energy E (see Sec. 2). $\mathcal{I}$ is the moment of inertia of the nucleus: $\sigma^2=\mathcal{I}t/\hbar^2$ is the spin cutoff factor, which regulates the level spin distribution; t is the thermodynamic temperature; a is a characteristic parameter related to the single-nucleon spacings near the Fermi energy. In the case of Eq. (1), t is related to the effective excitation energy by the law $U=at^2$; in the case of Eq. (2), by the law $U=at^2-t$.

The discrepancy between Eqs. (1) and (2) is due to the approximate mathematical procedure of the saddle-point method.[7,8] A straightforward comparison with exact combinatorial calculations based on the equidistant spacing model seems to indicate that Eq. (2) is a little more accurate for a two-fermion gas with equidistant nucleon spacings.[2,8,9] For this reason, in the following sections, we use Eq. (2) to fit the experimental level densities of light nuclei.

The comparison of theory with experiments is usually done by analyzing: (i) slow-neutron resonance data[4,10,11,7]; (ii) energy spectra of statistical reactions[10–12]; (iii) low-energy level distributions (up to $\sim$5–6 MeV excitation energy)[13,14]; (iv) level widths of nuclei.[15–17]

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Most analyses concern the behavior of level density with the mass number of excited nuclei. The experimental results show a smooth dependence of the parameter a, appearing in Eqs. (1) and (2), on the mass number A in the case of light nuclei ($A \leqslant 70$), and strong shell effects in the case of heavier nuclei.[10]

Other works test the energy dependence of $\rho(E,J)$. A considerable effort has been made,[17] in a work hereafter called I, to obtain from the analysis of experimental results a set of parameters that, with the help of Eq. (2), allows a correct estimation of the slope and absolute value of experimental level densities over a wide range of excitation energies (from ~6–7 up to ~20 MeV). The authors of I took only the light-nucleus region ($20 \leqslant A \leqslant 70$) into consideration, where the experimental level densities vary smoothly with the mass number A, and it seems quite plausible to apply a simple model like the equidistant spacing model. In this manner they deduced empirical parameters which in fact seem to fulfill the expected requirements.

Further work was, however, necessary: (I) to establish with more accuracy the value of the proposed parameters and the energy range of their validity; (II) to understand whether they are mere effective parameters, or have a physical meaning.

To answer point (I) we have thought it useful to reanalyze the great number of data concerning the spectra of statistical reactions induced on light nuclei ($A \leqslant 70$).

It was recently suggested[18] that the results of nearly all analyses are biased by the rough approximations one usually makes to simplify the work.

To facilitate the comparison of the results of our analysis with the analyses performed by other authors with different approaches, we have used both Eqs. (1) and (2) for extracting the empirical values of parameter a. In the case of Eq. (1) we have used the usual effective value[10,11] for the excitation energy of residual nucleus. In the case of Eq. (2) we have used both the usual effective value of the excitation energy of residual nucleus and the one suggested in paper I.

The experimental data used, the analysis method followed, and the results obtained will be shown in Sec. 2.

In Sec. 3, for the reason previously stated, we will compare the new results obtained by using Eq. (2) with those of paper I and those obtained by analyzing new slow-neutron resonance data.

Finally, in Sec. 4, we will discuss whether the conclusions we reach in Sec. 3 agree with the experimental distribution of low-energy levels observed with various reactions and we will make some comments about other analyses recently performed, concerning this subject.[7,14]

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2. ANALYSIS OF STATISTICAL REACTION SPECTRA

In recent years, many studies have established that (n,p), (n,n'), and (n,α) reactions induced by not too large energy neutrons on light nuclei are mostly compound-nucleus reactions.[10,12,19–21] Most of these reactions have been induced by 14-MeV neutrons.

Other studies of reactions induced by charged particles give results which are more difficult to interpret. We must note that in many cases, because of the high energy of the bombarding particles, the excitation energy of the intermediate system allows the emission of more than one particle; on the other hand, the high-energy part of the emitted-particle spectrum, which should correspond only to one-particle emissions, shows that direct processes are surely superimposed on the statistical ones.[22–24]

On the other hand, other reactions induced on light nuclei by particles of energy up to about 15 MeV are not very different from the neutron-induced ones and can also be mainly attributed to compound-nucleus processes.

Let us fix the general notation we will use: E_{inc} is the c.m.-system energy of initial channel, Q is the Q value of the reaction, ϵ is the final channel kinetic energy in the c.m. system, and U is the "effective" excitation energy of the residual nucleus.

To analyze the experimental results one usually makes some rough approximations which greatly simplify the exact expression (in the framework of the statistical model) that relates the spectrum $n(\epsilon)$ of emitted particles to the level density of the residual nucleus.

Assuming as infinitely large all the spin cutoff factors σ^2 appearing in the level densities of various residual nuclei to which the compound nucleus (C.N.) can decay, one obtains the following widely used expression:

$$n(\epsilon)d\epsilon \propto \epsilon\sigma_c(\epsilon)\rho(E,0)d\epsilon. \tag{3}$$

$\sigma_c(\epsilon)$ is the inverse cross section at energy ϵ.

In a previous work[18] we have shown that, in order to obtain more accurate results, one must take into account in a more realistic way the level spin distribution of nuclei to which the C.N. can decay. We suggested (i) for (n,n'), (n,p), (p,p'), (p,n), (α,p), and (α,n), reactions the following expression:

$$n(\epsilon)d\epsilon \propto \epsilon\sigma_c(\epsilon)\rho(E,0)\sigma^2 d\epsilon, \tag{4}$$

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where σ^2 is the energy-dependent spin cutoff factor of the residual nucleus; (ii) for (n,α) and (p,α) reactions the slightly different expression

$$n(\epsilon)d\epsilon \propto (\epsilon)^{1/2}\sigma_c(\epsilon)\rho(E,0)\sigma^2 d\epsilon. \quad (5)$$

For a careful discussion of this point see Ref. (18).

In this analysis we will use Eqs. (4) and (5).

These equations, combined with either level-density equations (1) or (2), allow one to obtain empirical values of the parameter a from the experimental spectra. One must only take into account that, if we assume Eq. (1),

$$\sigma^2=\frac{\mathcal{I}}{\hbar^2}\left(\frac{U}{a}\right)^{1/2}; \quad (6)$$

if we assume Eq. (2),

$$\sigma^2=\frac{\mathcal{I}}{\hbar^2}\left(\frac{U+t}{a}\right)^{1/2}, \quad (7)$$

where $\mathcal{I}$ is the moment of inertia of the residual nucleus.

As far as the excitation energy of the residual nucleus is concerned, most authors assume that U should be an "effective energy" which is obtained by subtracting from the excitation energy $E=E_{\text{inc}}+Q-\epsilon$ the pairing energy Δ (see Refs. 7 and 25):

$$U=E-\Delta. \quad (8)$$

In paper I it is suggested that, for light nuclei $(20\leqslant A\leqslant 70)$, U should be given by the expression

$$U=E-\Delta+(70/A \text{ MeV}), \quad (9)$$

where A is the mass number. The term $70/A$ MeV is an average term which shifts in a smooth way the zero of the energy scale of different excited nuclei.

As pointed out in the Introduction, we have analyzed the experimental spectra by utilizing both Eqs. (1) and (2) for the level densities and the two types of effective energies.

We will denote by a_1 the a values obtained by using Eq. (4) or (5), depending on the type of the reactions, and the level density $\rho(E,0)$ given by (1) with the effective excitation energy $U=E-\Delta$; a_2 the a values obtained by using Eq. (4) or (5) and the level density $\rho(E,0)$ given by (2) with the effective excitation energy $U=E-\Delta$; a_3 the a values obtained by using Eq. (4) or (5) and the level density $\rho(E,0)$ given by (2) with the effective excitation energy $U=E-\Delta+(70/A$ MeV).

The pairing energies Δ are those given by Cameron.[25]

Before showing the results obtained, let us discuss briefly the quantities $\sigma_c(\epsilon)$.

We have

$$\sigma_c(\epsilon)=\pi\lambda^2\sum_{l=0}^{l_{\max}}(2l+1)T_l(\epsilon). \quad (10)$$

The transmission coefficients $T_l(\epsilon)$ are calculated using optical-model potentials with parameters usually obtained by a best fit of elastic scattering differential cross sections.

In general, the agreement of optical-model calculations with experimental elastic cross sections is good for nuclei which are not too light and energies which are not too low. We apply these calculations both to light nuclei and to quite low energies of emitted particles.

However, this procedure should not introduce significant systematic errors, as we can conclude from the fact that we obtain quite good agreement between a values deduced by means of reactions leading to the same residual nuclei with emission of different particles.

For the evaluations of σ_c we have used, in the case of protons, the results of Ref. 26 (they do not differ sensitively from those of Perey and Lindner[27]); in the case of neutrons, the results of Ref. 28; and in the case of α particles, the results of Ref. 29. The a values we obtained are not characterized by a quoted error. This is due to the fact that we think it is very difficult to estimate these errors correctly because of the experimental uncertainties and analysis approximations. We think, however, that the average behavior of a values with A can be correctly deduced without being greatly affected by local fluctuations of single values.

The results we obtained are summarized in Table I.

From row I to row VIII are successively tabulated: the reactions analyzed, the initial channel energy in the c.m. system, the reference, the excitation energy range ΔE of the spectrum we analyzed, a_1, a_2, a_3, and the mass number of the residual nucleus. The a values are extracted by experimental spectra by means of a computer and a best-fit FORTRAN program, following the method outlined in Ref. 18. The high-excitation-energy limit of the spectra we analyzed is suitably chosen in order to eliminate the possibility of multiple particle emission. The choice of the low-excitation-energy limit is not performed according to a general law; it depends on various factors such as the reaction, the possible presence of direct effects, the observation angle, etc. In Table I, on the average, the low-excitation-energy limit is given by $E=\Delta+(1.5$ MeV), but in many cases it is considerably lower $[E\sim\Delta+(0.5$ MeV)].

The excitation energy range considered for each nucleus is, on the average, 5 MeV.

In the case of (n,α) and (p,α) reactions, whenever necessary, we have taken into account the angular dependence of the spectrum slope, and we have introduced a correction for this effect. (For a discussion of this point see Ref. 18.)

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TABLE I. a_1, a_2, a_3 values (see text) obtained by analyzing energy spectra of particles emitted in statistical reactions.

Reaction	E_{inc} (MeV)	Reference	Excitation energy range	a_1 (MeV^{-1})	a_2 (MeV^{-1})	a_3 (MeV^{-1})	A_R
$N^{14}(n,\alpha)B^{11}$	13.72	a	3.56–11.68	2.91	1.66	1.57	11
$Na^{23}(n,\alpha)F^{20}$	13.51	b	0.40– 6.92	2.65	1.99	2.25	20
$Al^{27}(n,\alpha)Na^{24}$	14.27	c	0.59– 5.29	2.76	2.01	2.11	24
$Al^{27}(n,\alpha)Na^{24}$	13.78	d	1.64– 6.66	3.65	3.22	4.08	24
$Mg^{25}(n,p)Na^{25}$	13.46	e	2.90– 5.41	3.54	2.70	3.21	25
$Al^{27}(n,p)Mg^{27}$	13.50	e	2.41– 7.91	4.05	3.20	3.88	27
$Al^{27}(n,n')Al^{27}$	6.75	f	2.49– 6.25	3.62	2.71	3.20	27
$Al^{27}(p,n)Si^{27}$	15.42	g	2.53– 7.70	4.26	3.52	4.53	27
$Si^{28}(n,p)Al^{28}$	13.61	h	0.62– 5.86	3.75	3.06	3.71	28
$P^{31}(n,\alpha)Al^{28}$	14.05	i	0.16– 6.10	3.75	2.73	3.03	28
$S^{32}(n,p)P^{32}$	13.58	e	2.16– 7.67	4.20	3.86	4.69	32
$Ca^{40}(n,\alpha)Ar^{37}$	13.76	b	4.59– 8.93	4.00	3.71	4.43	37
$Ca^{40}(n,\alpha)Ar^{37}$	14.15	i	3.32– 8.76	4.57	4.20	4.63	37
$Ca^{40}(n,p)K^{40}$	13.66	e	2.62– 6.60	6.34	5.99	7.25	40
$Ca^{40}(n,p)K^{40}$	14.14	j	1.72– 7.62	5.32	4.94	5.94	40
$Ar^{40}(\alpha,p)K^{43}$	14.50	k	2.20– 7.71	5.64	5.06	6.12	43
$Ar^{40}(\alpha,p)K^{43}$	16.01	k	2.62– 9.14	5.67	5.24	6.24	43
$Ti^{47}(n,p)Sc^{47}$	13.71	e	3.45– 8.05	5.47	5.11	5.95	47
$V^{51}(n,\alpha)Sc^{48}$	13.73	l	0.86– 5.15	6.40	5.73	7.11	48
$V^{51}(p,\alpha)Ti^{48}$	17.36	m	3.30– 9.88	5.35	4.55	5.22	48
$Cr^{50}(n,p)V^{50}$	13.72	e	5.47– 7.95	5.89	5.67	6.37	50
$V^{51}(n,n')V^{51}$	6.87	f	2.02– 6.35	5.96	5.22	6.24	51
$V^{51}(p,n)Cr^{51}$	7.85	n	2.11– 4.94	5.70	4.87	5.84	51
$Cr^{52}(n,p)V^{52}$	13.73	e	2.02– 5.02	7.20	6.75	8.08	52
$Fe^{56}(n,\alpha)Cr^{53}$	13.85	b	4.13– 7.67	5.52	5.17	5.96	53
$Fe^{56}(p,\alpha)Mn^{53}$	17.18	m	1.61– 9.20	5.45	4.49	5.06	53
$Fe^{54}(n,p)Mn^{54}$	13.56	o	3.69– 9.12 5.62– 9.12	5.73 7.74	5.48 7.53	6.15 8.41	54
$Fe^{54}(n,p)Mn^{54}$	13.74	e	2.82– 7.84	6.57	6.26	7.16	54
$Mn^{55}(n,n')Mn^{55}$	6.87	f	2.28– 6.34	5.37	4.79	5.57	55
$Ni^{58}(n,\alpha)Fe^{55}$	13.86	b	1.93– 9.80	5.91	5.31	6.29	55
$Mn^{55}(p,n)Fe^{55}$	7.86	n	2.03– 5.45	5.25	4.45	5.18	55
$Ni^{58}(p,\alpha)Co^{55}$	17.30	m	1.78– 6.99	5.25	4.37	4.93	55
$Fe^{56}(n,p)Mn^{56}$	13.53	o	2.40– 6.45	6.18	5.81	6.68	56
$Fe^{56}(n,p)Mn^{56}$	13.75	e	0.08– 7.07	6.74	4.93	5.49	56
$Co^{59}(n,\alpha)Mn^{56}$	13.76	d	2.07– 7.07	7.39	7.01	8.17	56
$Fe^{56}(n,n')Fe^{56}$	6.87	f	3.96– 6.38	5.74	5.06	6.00	56
$Fe^{56}(p,p')Fe^{56}$	9.82–10.31 10.80–11.78	p	4.26– 7.26	5.43	4.88	5.69	56
$Co^{59}(p,\alpha)Fe^{56}$	11.80 12.61 13.27	p	4.26– 7.56 8.77 9.27	4.51	4.03	4.57	56
$Co^{59}(p,\alpha)Fe^{56}$	17.30	m	4.65– 9.43	4.78	4.39	4.97	56
$Fe^{56}(\alpha,\alpha')Fe^{56}$	19.60	q	3.83–13.53	5.95	5.58	6.36	56
$Fe^{56}(p,n)Co^{56}$	15.71	g	3.46– 8.24	7.47	7.20	8.18	56
$Ni^{58}(n,p)Co^{58}$	13.76	e	2.62– 8.12	5.87	5.57	6.28	58
$Ni^{58}(n,p)Co^{58}$	13.76	r	2.82– 7.73	5.94	5.63	6.36	58
$Ni^{58}(n,p)Co^{58}$	14.55	s	1.93– 7.78 3.84– 7.78	6.47 7.85	6.11 7.59	7.02 8.59	58
$Ni^{58}(n,p)Co^{58}$	14.55	t	5.93– 9.61	5.83	5.63	6.18	58
$Mn^{55}(\alpha,n)Co^{58}$	13.05 15.84 18.64	u	3.10–11.09	6.04	5.81	6.43	58
$Ni^{58}(p,p')Ni^{58}$	11.21	v	4.82– 7.57	7.30	6.85	8.04	58
$Co^{59}(p,n)Ni^{59}$	7.86	n	1.95– 4.54	6.88	5.93	7.24	59
$Fe^{56}(\alpha,n)Ni^{59}$	13.06 15.86 18.66	u	1.66– 6.71	6.70	6.05	7.12	59
$Ni^{60}(n,p)Co^{60}$	13.50	o	3.17– 7.18	6.71	6.41	7.24	60
$Ni^{60}(n,p)Co^{60}$	13.77	e	2.75– 6.76	6.34	6.00	6.81	60

TABLE I (*continued*)

Reaction	E_{inc} (MeV)	Reference	Excitation energy range	a_1 (MeV^{-1})	a_2 (MeV^{-1})	a_3 (MeV^{-1})	A_R
$Cu^{63}(n,\alpha)Co^{60}$	13.78	w	3.84– 7.51	7.91	7.64	8.61	60
$Zn^{64}(n,\alpha)Ni^{61}$	13.88	b	3.19– 7.67	6.95	6.53	7.55	61
$Ni^{58}(\alpha,p)Cu^{61}$	11.97	v	1.66– 6.26	8.56	7.51	9.13	61
$Cu^{63}(n,p)Ni^{63}$	13.78	e	3.50– 8.50	6.06	5.69	6.45	63
$Cu^{63}(n,p)Ni^{63}$	13.78	r	4.94– 7.97	5.83	5.53	6.19	63
$Cu^{63}(n,n')Cu^{63}$	4.92	f	1.67– 4.43	7.58	6.22	7.49	63
$Cu^{63}(n,n')Cu^{63}$	6.89	f	2.48– 6.37	7.29	6.70	7.95	63
$Zn^{64}(n,p)Cu^{64}$	7.88	x	0.34– 4.33	8.03	6.97	8.33	64
$Zn^{64}(n,p)Cu^{64}$	13.78	e	2.99– 8.01	7.05	6.75	7.55	64
$Zn^{64}(n,p)Cu^{64}$	13.78	x	3.31– 8.76	6.28	6.02	6.69	64
$Zn^{66}(n,p)Cu^{66}$	13.79	e	2.43– 7.45	7.95	7.61	8.62	66
$Zn^{67}(n,p)Cu^{67}$	13.79	e	4.51– 9.47	10.13	9.83	11.12	67

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3. COMPARISON OF a VALUES DEDUCED FROM STATISTICAL SPECTRA, SLOW-NEUTRON RESONANCES, AND LEVEL WIDTHS

In this section, we will limit ourselves to discussing the results we obtained concerning the a_2 and a_3 values. We will compare these values with those obtained by analyzing slow-neutron resonances at $E\sim 8$ MeV, and those corresponding to a higher energy $E\sim 19$ MeV, extracted from level widths in the continuum energy region.

In paper I, a comparison between a values deduced by analyzing slow-neutron resonance data and level widths at high excitation energy, by using the level-density expression (2), was already made. Recently, however, Cameron[7] has reported a list of slow-neutron resonance spacings for light nuclei that in some cases are slightly different from those utilized in paper I. In Table II we give a new list of slow-neutron resonance spacings with their corresponding a_2 and a_3 values. To obtain the a_2 values reported in Table II, we utilized Eq. (2) for $\rho(E,J)$. For the moment of inertia we have taken the rigid-body value corresponding to a spherical nucleus, with radius $R\sim 1.27A^{1/3}$ F.

The effective excitation energy U of the compound nucleus was taken as $U=E-\Delta$.

E is either the neutron binding energy B or the "effective neutron binding" B_{eff}, given by the relation $B_{eff}=B+\bar{E}_{inc}$, where $\bar{E}_{inc}$ is the average value of the initial channel energy in the c.m. system and is of the order of 100 keV.

Hereafter we will call the following set of parameters set A:

$$a_2,\ \mathcal{I}=\mathcal{I}_{rig},\quad R\cong 1.27\,A^{1/3}\,\mathrm{F},\quad U=E-\Delta.$$

Likewise, to obtain the a_3 values, we utilized Eq. (2) for $\rho(E,J)$; as in paper I, for the moment of inertia we assumed the spherical-nucleus rigid-body value corresponding to a radius $R\sim 1.27A^{1/3}$ F.

The effective excitation energy U of the C.N. was taken as $U=(E-\Delta+(70/A)$ MeV). E is defined as in the previous case.

The a_2 values, reported in Table II, are not appreciably different from those one would obtain by analyzing the slow-neutron resonance spacings reported in Ref. 10 and utilized in paper I; the a_3 values are not appreciably different from those, corresponding to slow-neutron resonances, reported in Table III of paper I, and their average value is again roughly given by the law $a_3\sim(0.127A)$ MeV^{-1}.

This law giving the average a_3 values is immediately

TABLE II. Slow-neutron resonance spacings for light nuclei. (C.N. = compound nucleus; B = neutron binding energy; $B_{\text{eff}} = B + \bar{E}_{\text{inc}}$; D_{obs} = observed resonance spacing.)

Compound nucleus	Reference	B (MeV)	B_{eff} (MeV)	D_{obs} (KeV)	a_2 (MeV^{-1})	a_3 (MeV^{-1})
P^{32}	a	7.937		50	4.58	3.90
Cl^{36}	b	8.577		13.3±2.6	5.09	4.42
Cl^{38}	a	6.110		27.8	5.83	4.88
K^{40}	a	7.798		11.2	5.74	4.98
	b		7.85	10±2	5.83	5.06
Ca^{41}	b	8.361	8.6	50±5	5.90	5.12
K^{42}	b	7.530		10±2	6.06	5.28
Ca^{44}	b	11.136		4±0.8	6.41	5.60
Sc^{46}	b	8.766		2.2±0.3	7.25	6.46
Ti^{47}	b	8.887	9.05	28±4	6.55	5.78
V^{51}	a	11.040		1.26	7.16	6.36
V^{52}	b	7.304		3.7±0.4	7.30	6.42
Cr^{53}	b	7.943	8.2	44±6	6.76	6.00
Fe^{55}	b	9.300	9.5	25±3	6.55	6.01
Mn^{56}	b	7.270		2.1±0.7	8.01	7.13
Fe^{57}	b	7.641	7.9	29±3	7.55	6.71
Ni^{59}	a	9.001		25.7	6.92	6.26
	b		9.1	24±3	6.97	6.31
Co^{60}	b	7.497		2.5±0.5	7.74	6.94
Ni^{61}	a	7.873		22	7.99	7.15
	b		7.9	21±3	8.04	7.18
Cu^{64}	b	7.916		1±0.15	8.74	7.94
Zn^{65}	a	7.989		1.85	10.42	9.37
Cu^{66}	a	7.061		1.99	8.84	7.96
	b			1.7±0.4	9.00	8.12
Zn^{67}	b	7.046		6.5±1.0	10.12	9.02
Zn^{68}	a	10.199		0.69	9.25	8.40
Ga^{70}	a	7.730		0.30	10.36	9.44

[a] E. Erba, U. Facchini, and E. Saetta Menichella, Nuovo Cimento 22, 1237 (1961).
[b] A. Gilbert and A. G. W. Cameron, Can. J. Phys. 43, 1446 (1965).

interpretable in the framework of the Fermi-gas model, and it corresponds to a nuclear radius $R \sim 1.5A^{1/3}$ F. (The Fermi-gas model predicts $a = kA$; k is given by the law $k = \pi^2/4\epsilon_0$; ϵ_0 is the Fermi energy.[6])

As in paper I, we explain the lower value of the moment-of-inertia radius as due to a reduction of the moment of inertia to ~70% of the rigid-body value ($\mathcal{I}_{\text{rig}} = \frac{2}{5}AR^2$) corresponding to a spherical nucleus with $R \sim 1.5A^{1/3}$ F. (For a discussion of this point see paper I.)

The following set of parameters: a_3, $\mathcal{I} = 0.7\mathcal{I}_{\text{rig}}$, $R \sim 1.5A^{1/3}$ F, $U = E - \Delta + (70/A$ MeV), will be called set B.

In Fig. 1 we plot against the mass number A the a_2 values obtained by analyzing statistical reaction spectra (open circles) and slow-neutron resonances (black circles). In this figure we do not explicitly give the a_2 values which could be deduced from level widths at high excitation energy; however, we have drawn the straight line $a \sim 0.127A$ MeV^{-1} that represents with a good approximation the average behavior of these values with the mass number A. (The a values obtained by analyzing the level widths correspond to nuclei with mass numbers ranging from $A \sim 24$ to $A \sim 58$.)

As one can see, the a_2's deduced from slow-neutron resonances are larger than the corresponding a_2's obtained by means of level widths, which in turn are, on the average, larger than the a_2's obtained from statistical spectra.

The disagreement between the a_2 values deduced from slow-neutron and level-width data is carefully discussed in paper I, and it suggested the introduction of the shift of the effective-excitation-energy scale we used to calculate the a_3 values.

The disagreement between the a_2 values from slow-neutron and statistical-spectrum data, though they correspond to not very different excitation energies, can be explained as follows: The slow-neutron resonance data fix the absolute value of $\rho(E,J)$ to an energy $E \sim 8$ MeV; the statistical spectra data give the slope of the level density. The disagreement means that, in the energy region $(1.5+\Delta) \lesssim E \lesssim (6.5+\Delta)$ MeV, the slope of the level density (2) with set A of parameters, that correctly give the experimental absolute value at $E \sim 8$ MeV, is bigger than the experimental one. This fact is in agreement with the conclusions reported in paper I; there it was concluded that the theoretical level density (2), with the parameters of set A that

correctly give the absolute value at $E\sim 8$ MeV, grows too rapidly with respect to the experimental one going from $E\sim 8$ to $E\sim 19$ MeV.

The shift in the effective energy scale was introduced in order to obtain, with the same a values which correctly give the level density at $E\sim 19$ MeV, the correct absolute value also at the lower energy $E\sim 8$ MeV. Consequently, $\rho(E,J)$, with set B of parameters suggested in paper I and here introduced to evaluate a_3, has a lower slope. This can be seen in Fig. 2, where we compare, as a function of $E-\Delta$, for a nucleus with a mass number $A=45$, $\rho(E,\frac{1}{2})$ given by (2) and set A of parameters ($a_2=6.65$ MeV^{-1}) with $\rho(E,\frac{1}{2})$ given by (2) and set B of parameters ($a_3=5.715$ MeV^{-1}).

An important observation must be made: In Fig. 1, the straight line that gives the average behavior of the a_2 values deduced from level widths is obtained by utilizing, in the level-width expression, the slow-neutron resonance data to estimate the absolute value of the level density of the residual nuclei to which the C.N. can decay. (For a description of the method of analysis of level-width data see paper I.)

The a_2 values obtained by analyzing statistical spectra are in poor agreement with those fitted by this straight line. This agreement could, however, be much improved by using the a's obtained from statistical spectra to estimate the absolute value of the level density of the residual nuclei in the level-width expression. In this case, one would obtain, from the level widths, a_2 values lower than those fitted by the straight line of Fig. 1. This procedure does not seem justified by the experimental results because, in the level-width expression, the absolute values of the calculated residual-nucleus level densities would disagree with the ones we obtain from slow-neutron resonances.

However, we must emphasize the great importance, in our procedure, of slow-neutron data, which are the only experimental results we have considered that unambiguously fix the absolute value of level density at a given energy.

In Fig. 3 we compare the a_3 values obtained from statistical spectra (open circles) with those obtained by analyzing slow-neutron resonances (black circles) and level widths. In this figure, we do not report explicitly the a_3 values deduced from level widths at high excitation energy: We have simply drawn the straight line, suggested in paper I, that with very good accuracy gives the average behavior of their value with A. One can note the very good agreement of the average behavior of the a_3 values extracted by the three different types of experimental results.

Figure 3 of this paper and Fig. 2 of paper I indicate that single a_3 values deduced from statistical spectra fluctuate, from nucleus to nucleus, somewhat more than the ones obtained by analyzing slow-neutron resonances and level widths. These fluctuations include contributions from the experimental and theoretical uncertainties affecting the analysis of statistical spectra.

The good agreement between the a_3 values from different experimental results confirms the reliability of set B of level-density parameters ($a_3\sim 0.127A$ MeV^{-1}) and, as can be seen in Table I, extends the range of their validity down to excitation energies as low as $E\sim\Delta+(1$ MeV).

To eliminate any doubts about the reliability of the slow-neutron data, reported before, we think it would be very useful to evaluate the absolute value of statistical cross sections to single final levels and compare the theoretical results obtained with the expression (2) for the level density and set B of parameters, on the one hand, with the experimental results on the other.

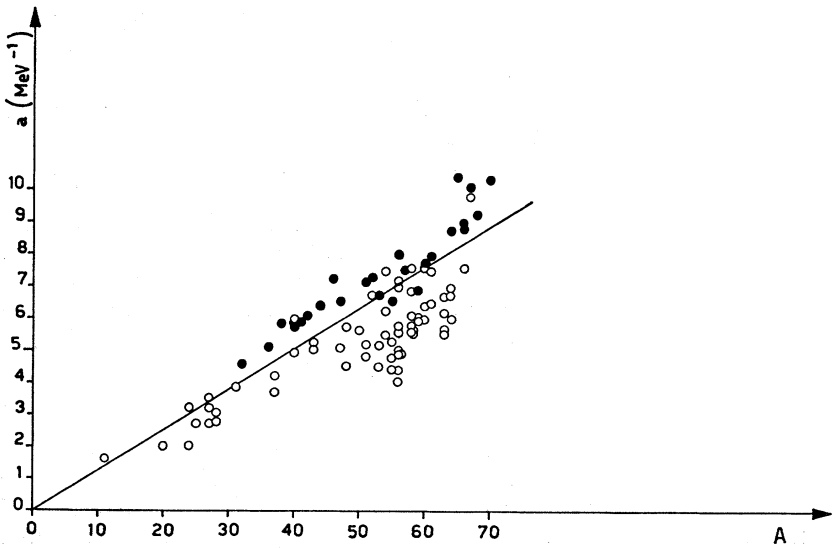

FIG. 1. A plot against mass number A of the a_2 values obtained by analyzing statistical reaction spectra (open circles) and slow-neutron resonances (black circles). The straight line represents, with a good approximation, the average behavior of the a_2 values extracted by level widths at high excitation energy.

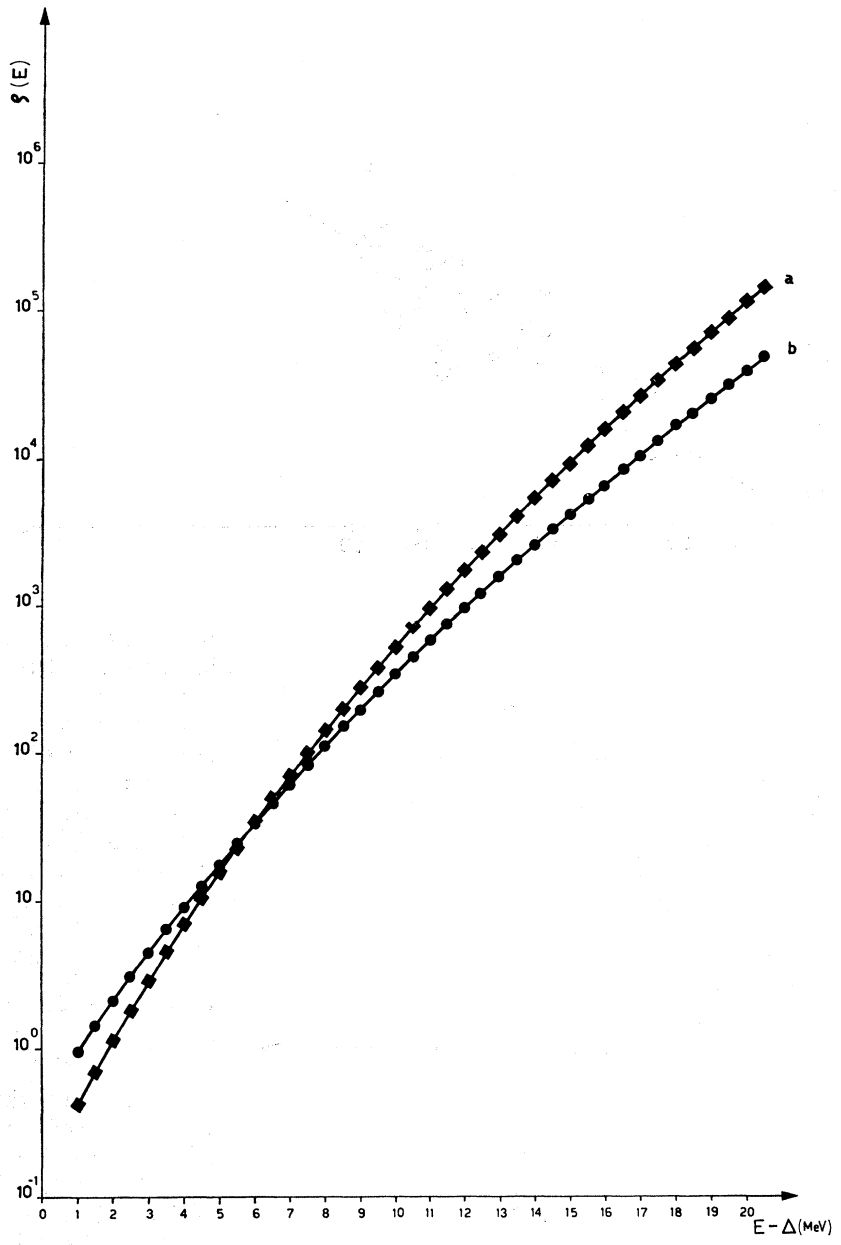

FIG. 2. Behavior with the energy of the level density $\rho(E,\frac{1}{2})$ given by Eq. (2), in the case of an $A=45$ nucleus: (a) utilizing set A of parameters ($a_2=6.65$ MeV^{-1}); (b) utilizing set B of parameters ($a_3=5.715$ MeV^{-1}).

4. LOW-ENERGY LEVELS OF LIGHT NUCLEI. DISCUSSION OF EXPERIMENTAL RESULTS AND COMPARISON WITH THEORETICAL CALCULATIONS

We must now discuss the results, referring to low excitation energies obtained by a different method, e.g., counting the levels of residual nuclei in charged-particle reactions.

This method was first suggested by Ericson[13] and recently applied by Cameron to a wide variety of light nuclei.[14]

In particular Cameron reaches the conclusion that the level density of nuclei at low excitation energy cannot be represented by a Fermi-gas-type law, but that a law like the following must be used:

$$\rho(E,J)\propto A(2J+1)\exp[-J(J+1)/2\sigma^2]\exp(E/T). \tag{11}$$

At an energy that sometimes is lower, sometimes higher than the neutron binding energy, there will be a matching point which connects Eq. (11) to Eq. (1).[7]

Apart from the difference between Eqs. (1) and (2) used by us, the conclusions of Cameron are very different from those we reached in Sec. 3. We want now to discuss this point, and also, without making a fully detailed analysis, we will show that the experimental

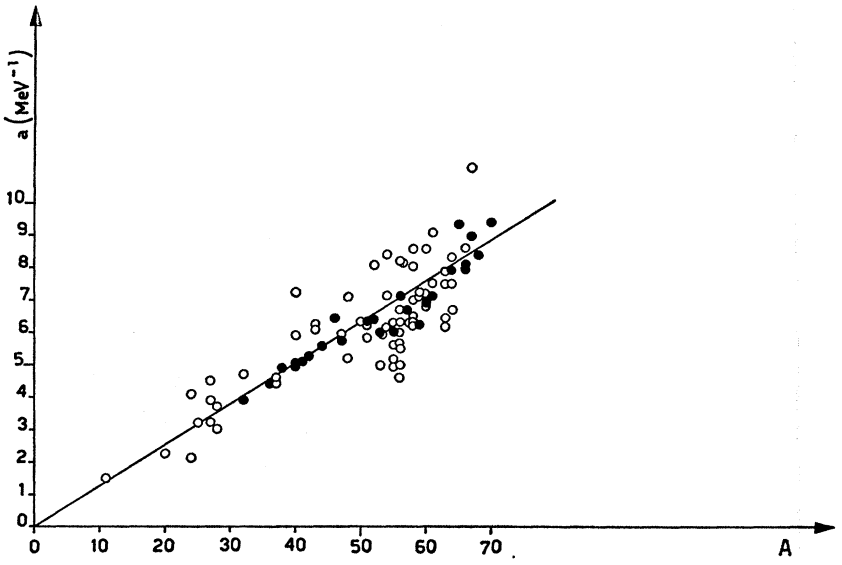

FIG. 3. A plot against mass number A of the a_3 values obtained by analyzing statistical reaction spectra (open circles) and slow neutron resonances (black circles). The straight line represents, with a good approximation, the average behavior of the a_3 values extracted by level widths at high excitation energy.

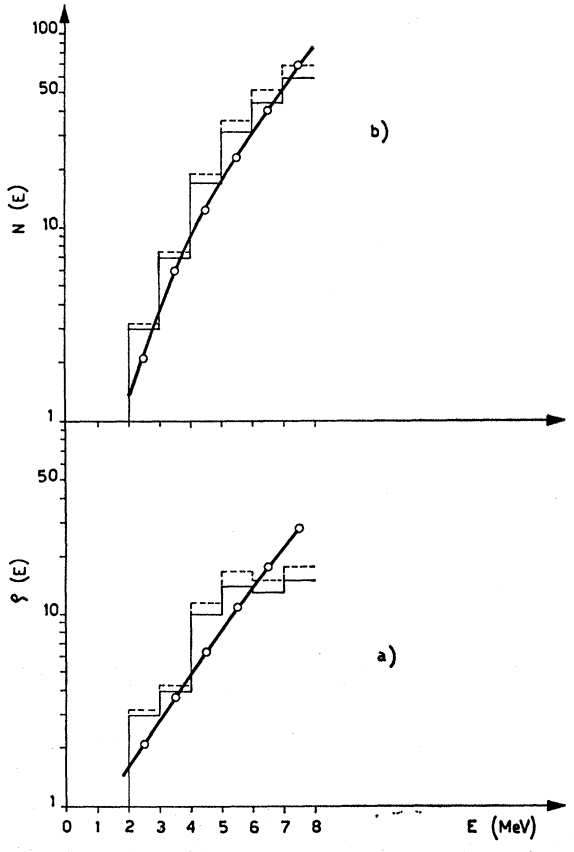

FIG. 4. Comparison between: (a) the experimental level density $\rho(E)$ of Mg^{25}, estimated by counting low excitation levels, and the theoretical one (full line) calculated by using (2) and set B of parameters; (b) the experimental cumulative number $N(E)$ defined in the text and the theoretical one (full line). The full-line histograms are obtained by counting the reported levels without correction for loss of levels due to finite energy resolution; the dashed-line histograms are obtained by introducing this correction.

results concerning low-excitation-energy levels are not in disagreement with our conclusions in Sec. 3.

First of all, we must make a careful discussion of the nuclear reactions we could utilize to obtain the final levels, whose distribution gives information on the behavior of the level densities.

(i) Some reactions, when the incident particle energy is not too high, proceed at least partially through the formation of a C.N. This, for instance, seems to be the case for (d,α),[30] (p,α),[31] (p,p'),[32] and (p,n)[33] reactions.

In many cases it is easy to show, with the help of optical-model calculations of transmission coefficients, that, especially when the target spin is high, almost all the levels of the residual nucleus are excited, and almost all the possible spins and parities are present (eventually a loss of high-spin levels will be possible). This happens because the C.N. reactions are not affected by the presence of configuration selection rules, and strict selection rules due to spin and parity conservation laws are not present. This seems to be the case of the levels of Mg^{25} obtained by means of the reaction $Al^{27}(d,\alpha)$-Mg^{25},[34] Ar^{38} obtained by means of the reaction K^{41}-$(p,\alpha)Ar^{38}$,[35] and Fe^{56} obtained by means of the reactions

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$Fe^{56}(p,p')Fe^{56}$ and $Co^{59}(p,\alpha)Fe^{56}$.[36] In Figs. 4–6 are quoted the results concerning such nuclei and the theoretical estimations (full line) obtained with the help of (2) and set B of parameters ($a_8=0.127A$ MeV^{-1}).

In these figures are given both

$$\rho(E)=\sum_J \rho(E,J) \quad \text{and} \quad N(E)=\int_{E_0}^{E}\rho(E')dE'$$

(in the case of odd-odd nuclei $E_0=0$, and in the case of even-odd and even-even nuclei $E_0=\Delta$, where Δ is the corresponding pairing energy[25]).

As one can see, the agreement is excellent also at very low excitation energies, confirming fully the results of Sec. 3. (A best-fit procedure with a very slight variation of such parameters could give a slightly better fit.)

We quote here only few of the possible experimental results. A very good agreement between experimental results and the level density (2) with set B of parameters is, for instance, obtained by analyzing low-energy levels of the following nuclei: Al^{26},[37–39] Ar^{37},[40] Sc^{48},[41,42] Ti^{48},[43] Cr^{50}, Cr^{52}, Cr^{53}, Cr^{54},[44] and Fe^{54}.[45] So far we have not made a systematic search of all the possible experimental results in this field; we can, however, affirm that, up to now, analyzing low-energy levels of residual nuclei obtained with reactions, at least partially of C.N. type, we have not found examples of meaningful

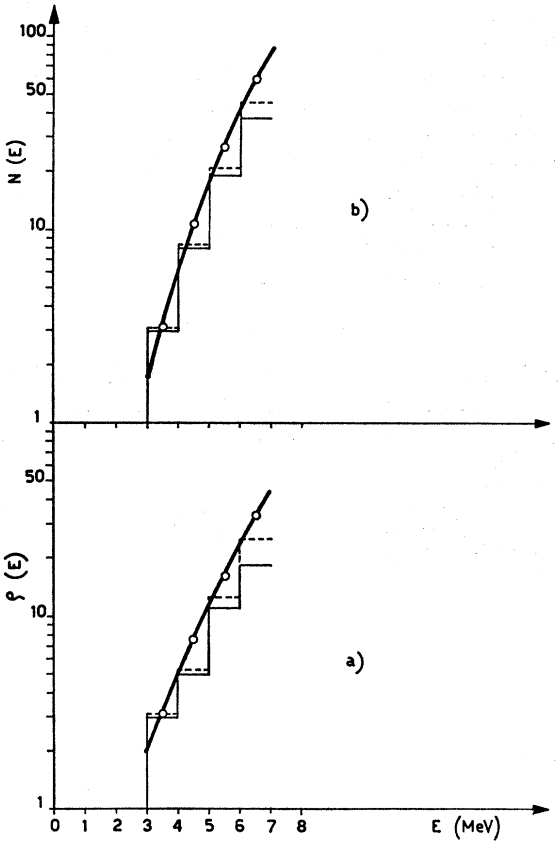

FIG. 5. Comparison between: (a) the experimental level density $\rho(E)$ of Ar^{38} estimated by counting low excitation levels, and the theoretical one (full line) calculated by using (2) and set B of parameters; (b) the experimental cumulative number $N(E)$ defined in the text and the theoretical one (full line). The full-line histograms are obtained by counting the reported levels without correction for loss of levels due to the finite energy resolution; the dashed-line histograms are obtained by introducing this correction.

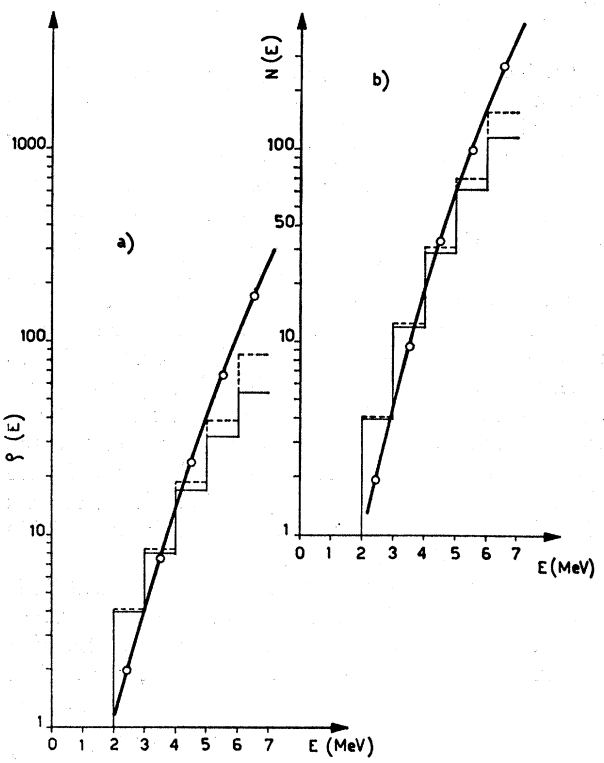

FIG. 6. Comparison between: (a) the experimental level density $\rho(E)$ of Fe^{56} estimated by counting low excitation levels, and the theoretical one (full line) calculated by using Eq. (2) and set B of parameters; (b) the experimental cumulative number $N(E)$ defined in the text and the theoretical one (full line). The full-line histograms are obtained by counting the reported levels without correction for loss of levels due to the finite energy resolution; the dashed-line histograms are obtained by introducing this correction.

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[39] In this case the levels of Al^{26} have been observed as final levels of two different reactions: $Mg^{24}(He^3,p)Al^{26}$ and $Al^{27}(He^3,\alpha)Al^{26}$.

[40] J. H. McNally, Nucl. Phys. **88**, 257 (1966).

[41] W. R. McMurray, M. Peisach, R. Pretorius, P. van der Merwe, and I. J. Van Heerden, Nucl. Phys. **A99**, 6 (1967).

[42] In the case of Sc^{48}, whose levels are obtained by means of the reaction $Ca^{48}(p,n)Sc^{48}$, it is easy to show that, in view of the transmission coefficients corresponding to incident and outgoing particles, only levels with $0\leqslant J\leqslant 3$ are observed, on the average.

[43] W. E. Dorenbusch, O. Hansen, D. J. Pullen, T. A. Belote, and G. Sidenius, Nucl. Phys. **81**, 390 (1966).

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[45] A. Aspinall, G. Brown, and S. E. Warren, Nucl. Phys. **46**, 33 (1963).

disagreement between our calculations and experimental results.

(ii) To other reactions, on the contrary, the contribution of C.N. processes is much lower. This seems to be, for instance, the case of (d,p) and (He^3,d) reactions, at quite high energy of incident particles.

As is well known, these reactions are mainly of the direct type. Apart from the spin- and parity-conservation laws, these reactions are restricted by selection rules due to the shell-model configuration of the target and the final nuclei. This happens because, in most cases, in a (d,p) or a (He^3,d) reaction the final nucleus appears in a shell-model configuration corresponding to a neutron or a proton captured in a single-particle state over a core corresponding to the target nucleus. Then the excited states of residual nuclei correspond to the excitation of the outermost nucleon. It is easy to deduce that, in general, many of the possible excitation states of residual nuclei are not reached with appreciable probability in these reactions.

As a clear demonstration of this fact we will limit ourselves to recalling the case of the reactions[46,47] $Ca^{42}(d,p)Ca^{43}$ already discussed by French.[48] Looking at Fig. 1 of Ref. 47, it is easy to note that, in a typical experiment, the very small peaks corresponding to the excitations which are different from those just described are usually not counted. In this experiment the most prominent proton groups correspond mainly to a neutron captured on $l=1$ orbits, and the corresponding levels of residual nucleus have $J=\frac{1}{2}$ or $J=\frac{3}{2}$ and odd parity.

Another striking example of the loss of final levels in (d,p) experiments is given by Fig. 5 of Ref. (44), where a comparison is made between the cumulative numbers $N(E)$ of levels of Cr^{53} and Cr^{54} obtained by means of (d,p) and (p,p') reactions [note that the nuclear barrier acts much more strongly on the outgoing protons of (p,p') reactions, because of their much lower energy].

The examples seem to indicate that, in general, residual levels obtained with (d,p) or (He^3,d) reactions should be considered with caution when one deduces information from them about the behavior of the level densities. This is unfortunate because of the great number of (d,p) and (He^3,d) experiments on light target nuclei.

Apart from some approximations introduced by Gilbert, Chen, and Cameron[14] that are not completely justified at low excitation energy, the previous discussion seems to explain the disagreement between our conclusions and theirs. These authors have based their analysis over many nuclei whose levels have been obtained by means of (d,p) reactions. This is the case of the nucleus Cl^{36} which is discussed by them in detail.[37,49]

Looking at Figs. 4–6 we can say that the cumulative number $N(E)$ of low-energy levels can also be represented by a simple law, $N(E) \propto A \exp(E/T)$, as already deduced by Ericson.[13] No careful attempts, however, have been made so far to see if a level density $\rho(E) \propto A' \exp(E/T)$ could explain the level-density features of many nuclei over a large excitation energy interval.

This work and the previous paper I, seem, on the contrary, to establish the fact that the level densities of light nuclei, on the average, can be well described by the Eq. (2) with set B of parameters, from very low excitation energies up to $\sim$20 MeV.

ACKNOWLEDGMENTS

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