as the criterion, and this was measured from the strong satellites of the origin. For Ho(A) the angular separation of the two satellites at 1.093 Å was 4.54°, and for Ho(B), 3.71°, and the resolution was such that the two values could easily be distinguished.

A first sample of doubly distilled high-purity holmium containing 0.17% O_2 was studied in the form of massive pieces broken from the bulk specimen. A value of $\omega=30.7°$ was obtained at 4.2°K. A second sample of ordinary commercial 99+% metal, also containing 0.17% O_2 was studied in the form of unannealed filings. The lines were slightly broadened in comparison to those from the high-purity specimen. A value for $\omega=30.3°$ was deduced. The third powder sample was of commercial Ho in the form of filings and "sawdust" which had been in the laboratory in an unsealed container for over four years. This should have been a good sample of poor metal, but it too yielded a value of $\omega=30.6°$ at low temperatures. The three powder samples, as well as the other two single crystals Ho(C) and Ho(D) all had values of ω_f equal, within experimental error, to that of Ho(B).

It appeared that the unique behavior of Ho(A) could not be attributed to impurities in the metal, and a second possibility related to the manner of formation of the crystals was considered. The crystals Ho(B), Ho(C), and Ho(D) were all grown by a strain-anneal method whereas Ho(A) was cut from an as-cast, machined rod. Short of the obvious experiment of annealing Ho(A) (we are reluctant to risk losing a crystal which is by chance "frozen in" at a final turn angle of 36.7°) we have tried to study the effects of mechanical history by studying severely cold-worked polycrystalline specimens. It is not certain that the cold-worked state is necessarily equivalent to the as-cast condition of Ho(A), but in any case no effects of cold work on the final turn angle were observed. We still have no explanation for the properties of Ho(A).

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Quantum Oscillations of the Ultrasonic Absorption in Magnesium and Zinc*

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Quantum oscillations of the ultrasonic absorption have been studied in magnesium and zinc at magnetic fields between 5 and 20 kG. Low-amplitude oscillations, similar to the de Haas–van Alphen oscillations of magnetic susceptibility in period, temperature dependence of the amplitude, and line shape, are observed when $ql\sim1$, where q is the propagation constant of the sound wave and l is the mean free path of the electrons. The dependence of the amplitude on the angle between the magnetic-field vector and the direction of sound propagation does not agree with predictions. Oscillations of much larger amplitude have been observed in zinc with $ql\sim15$. Difficulty arises in comparing measurements of these oscillations with theory because they are characteristic of a region intermediate between the de Haas–van Alphen and giant oscillation regimes. Spin splitting of the absorption peaks is observed in zinc, but not in magnesium.

INTRODUCTION

QUANTUM oscillations of the acoustic-attenuation coefficient which are periodic in the reciprocal of the applied magnetic field have been observed in bismuth,[1–3] zinc,[4,5] antimony,[6] and gallium.[7] The phenomena are observed at high magnetic fields and are of two types: oscillations similar to de Haas–van Alphen oscillations in the magnetic susceptibility, and "giant quantum oscillations," which are a large amplitude resonance effect in the electron-phonon interaction. The factors determining which oscillations are observed are the sound frequency and mean free path of the electrons. de Haas–van Alphen oscillations occur when the frequency is low and the mean free path is short, and giant oscillations occur if the frequency is high and the mean free path is very long.

Although phenomena observed in bismuth, zinc, and gallium have been described in the literature as giant quantum oscillations, the electron mean free path must be nearly a centimeter for this effect, and only in gallium has this requirement been clearly satisfied. Close examination of magnetoacoustic oscillations reported in bismuth and zinc reveals that they are characteristic of some regime intermediate between the de Haas–van Alphen and giant quantum oscillation

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[6] J. B. Ketterson, Phys. Rev. **129**, 18 (1963).
[7] Y. Shapira and B. Lax, Phys. Rev. **138**, A1191 (1965).

regimes. The line shape of the absorption peaks is considerably broader than giant quantum oscillations and their amplitude is smaller.

The purpose of this paper is to report oscillations of the de Haas–van Alphen type observed in magnesium and zinc, and oscillations observed in zinc characteristic of an intermediate regime. The periodicity, amplitude, temperature dependence of the amplitude, spin-splitting, and other characteristics are discussed.

THEORY

In the giant-oscillation regime the sound-absorption mechanism may be treated as an electron-phonon interaction in which energy and momentum are conserved. This treatment is valid when the condition $\omega\tau\gg1$ is satisfied, where ω is the sound frequency and τ is the relaxation time of the electron assembly. When this condition is satisfied, uncertainty of the electron energy, according to the uncertainty principle and the finite lifetime introduced by scattering (described by τ), is much less than the phonon energy. Actually the lifetimes of the individual electron states are functions of energy and momentum; however, the value of $\omega\tau$ is a rough guide to the phenomena which are observed.

In order that uncertainty of the electron momentum be much less than the phonon momentum, the condition $ql\gg1$ must be satisfied, where q is the sound wave vector and l is the electron mean free path. Since $ql/\omega\tau=v_f/v_s$, where v_f and v_s are the Fermi and sound velocities, and $v_f/v_s\sim10^3$, this condition is satisfied for intermediate regimes where $\omega\tau<1$, as well as for the giant-quantum-oscillation regime. The de Haas–van Alphen regime occurs when $ql\lesssim1$ and, of course, $\omega\tau\ll1$.

In all regimes the absorption of sound in sufficiently high magnetic fields is a period function of the magnetic field **H**. The condition for absorption to occur is satisfied as successive Landau cylinders appear on the Fermi surface at a plane where k_z, the component of the electron wave vector in the direction of **H**, is given by

$$k_z^0=m^*v_sq/\hbar q_z=m^*v_s/\hbar\cos\theta. \tag{1}$$

θ is the angle between the direction of sound propagation and the magnetic field. The period is given by

$$\Delta(1/H)=2\pi e/\hbar cA_0, \tag{2}$$

where A_0 is the area of intersection of the k_z^0 plane with the Fermi surface. This expression is similar to that for the familiar de Haas–van Alphen period

$$\Delta(1/H)=2\pi e/\hbar cA_{\rm ex}, \tag{3}$$

except that the extremal area of the Fermi surface has been replaced by A_0. The periods will only be markedly different when $\cos\theta\ll1$, that is, when the direction of sound propagation is almost exactly perpendicular to **H**.

Expressions for the acoustic-attenuation coefficient in the giant-quantum-oscillation regime where electron scattering may be ignored ($\omega\tau\gg1$) were first derived by Gurevich, Skobov, and Firsov.[8] They predict that the amplitude of each absorption peak should be proportional to the magnetic field at which the peak occurs, to the sound frequency, and inversely proportional to the temperature; that is,

$$A_i\propto H_i\omega/T \tag{4}$$

for the ith peak. This relationship has been stressed by Shapira and Lax[7] and is confirmed by their results in gallium.

The effect on the sound-absorption mechanism of electron scattering by impurities and defects is an important consideration, since the oscillations change markedly as the mean free path and relaxation time are reduced. Also most experimental results published to date, as stated earlier, do not characterize the giant-oscillation regime where scattering is negligible. The effect of electron scattering in the electron-phonon interaction is to relax energy and momentum conservation. Thus the condition expressed by Eq. (1) specifying k_z^0 and those electrons which participate in the absorption is relaxed, and electrons with a range of k_z rather than those with k_z^0 alone will participate in the absorption. Applying the uncertainty principle, the level width and uncertainty of the z component of the momentum of the electron states are given approximately by $\Delta E\sim\hbar\omega/\omega\tau$ and $\Delta(\hbar k_z)\sim\hbar q_z/q_zl_z$, where $\hbar\omega$ is the phonon energy and $\hbar q_z$ is the component of phonon momentum in the direction of **H**. Since $q_zl_z=\omega\tau$ (from $l_z=v_z^0\tau$ and $v_z^0=v_s/\cos\theta$) electrons in a k_z range given approximately by

$$k_z^{\pm}=k_z^0\pm2k_z^0/\omega\tau \tag{5}$$

will participate in the absorption. The absorption peaks are thus broadened and reduced in amplitude by the electron scattering. In addition, if the range specified by Eq. (5) is large enough, electrons of more than one Landau level may participate in the absorption simultaneously. Indeed, if $\omega\tau\ll1$ and also $ql\ll1$, no restriction is imposed on the electrons that may participate in the absorption by energy and momentum conservation in the electron-phonon interaction. All electrons near the Fermi surface will participate and the absorption peaks will have the same periodicity as the de Haas–van Alphen effect.

Skobov[9] and Liu and Toxen[10] have derived an expression for the attenuation coefficient that holds when $(E_f/\hbar\omega_c)\gg(ql)^2\gg1$, where E_f is the Fermi energy

[8] V. L. Gurevich, V. G. Skobov, and Yu. A. Firsov, Zh. Eksperim i Teor. Fiz. **40**, 786 (1961) [English transl.: Soviet Phys.—JETP **13**, 552 (1961)].

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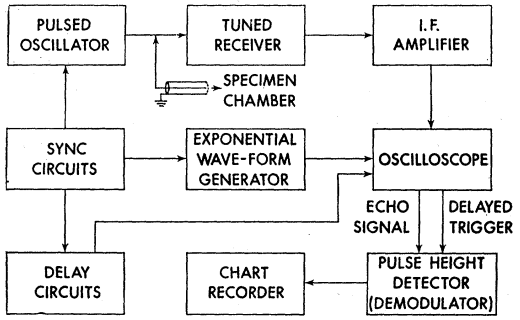

FIG. 1. Block diagram of ultrasonic circuitry.

and ω_c is the cyclotron frequency. They obtain

$$\Gamma(H)=\Gamma_0\Big[1+q_z l\frac{(\hbar\omega_c)^{1/2}}{E_f}\times\sum_{n=1}^{\infty}\frac{(-1)^n(2\pi n^{1/2}kT/\hbar\omega_c)\cos(2\pi nE_f/\hbar\omega_c-\pi/4)}{\sinh(2\pi^2 nkT/\hbar\omega_c)}\Big],\tag{6}$$

where Γ_0 is the zero-field attenuation. Thus the ratio of the amplitude of the oscillation to the zero-field attenuation is given by $(\hbar\omega_c/E_f)^{1/2}ql/\pi$ when $kT\ll\hbar\omega_c$. This expression describes a regime in which many Landau levels participate in the absorption and Eq. (6) is similar in form to expressions for de Haas–van Alphen oscillations of the magnetic susceptibility. An expression of similar form can be expected to apply when $q_z l\sim 1$.

Quinn and Rodriguez[11] have calculated the magnetoconductivity tensor, which oscillates with the magnetic field due to oscillations of the electron density of states. It should reflect the ultrasonic attenuation due to the electrons in the $\omega\tau\ll 1$, $ql\ll 1$ regime where the resonance factor is relatively constant. Perhaps this region, rather than $ql\lesssim 1$, is properly the de Haas–van Alphen region, but when $ql\sim 1$ the number of Landau levels participating in the ultrasonic attenuation is uaually large (for magnetic fields less than 30 kG) and the character of the oscillations is similar to the de Haas–van Alphen oscillations of magnetic susceptibility. When the ultrasound is propagated parallel to the magnetic field Quinn and Rodriguez find that the oscillatory part of the attenuation is approximately $(\hbar\omega_c/E_f)^{5/2}$ of the zero-field attenuation, and when they are perpendicular the ratio is $(\hbar\omega_c/E_f)^{3/2}$. Expressions derived by Skobov for $(E_f/\hbar\omega_c)\gg(ql)^2\gg 1$ are considerably different from the calculations of Quinn and Rodriguez for $\omega\tau\ll 1$.

EXPERIMENTAL TECHNIQUE

Magnesium specimens were prepared from single crystals grown by the molten zone process. The starting material was 99.999% pure magnesium supplied by American Mining and Smelting Company. Zinc specimens were prepared from single crystals of zinc (99.9999% pure) supplied by Consolidated Mining and Smelting Company and by Virginia Crystals Incorporated. Single crystals were annealed, oriented by x-ray diffraction, and spark-machined to specimens appropriate for ultrasonic absorption measurements. Reflecting surfaces for the ultrasound were hand-lapped parallel to within 30 sec of arc.

The specimens were mounted in a helium cryostat capable of operating at temperatures as low as 1.25°K. The Dewar system of the cryostat fitted between the pole pieces of an 8-in. magnet which was rotatable about a vertical axis. The magnetic field was measured with an accuracy of one part in 10^3 by a Hall probe.

A block diagram of the ultrasonic circuitry is shown in Fig. 1. Pulses of the required frequency and duration were generated by a pulsed rf oscillator and converted to ultrasound by a quartz transducer bonded to one of the parallel specimen surfaces. Silicone grease was used as the bonding agent. Echoes in the specimen were picked up by the same transducer and the resulting electronic pulses were transmitted to a tuned receiver and an i.f. amplifier which demodulated and amplified them. An electronic gate and appropriate circuitry selected one of the pulses and provided a continuous output proportional to the pulse amplitude. This output was displayed simultaneously with a signal from the Hall probe on a 2-pen chart recorder. The magnetic field was automatically varied linearly or reciprocally with time and the echo amplitude displayed continuously as a function of time.

In all experiments the quartz transducers were X cut and since, in all instances, the propagation direction was nearly parallel to a symmetry axis of the specimen the ultrasound was longitudinal.

Experiments were carried out on several single-crystal magnesium specimens; however, quantum oscillations in the ultrasonic absorption were only observed

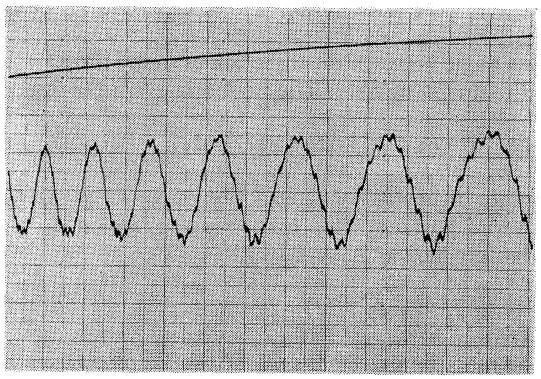

FIG. 2. Chart recording showing oscillations of the acoustic attenuation in magnesium. The magnetic field is in the (11$\bar{2}$0) plane, 20° from the (0001) axis, and is approximately 20 kG when the smooth Hall-probe trace is full scale. (The oscillatory trace monitoring the pulse amplitude is not zeroed.)

[11] J. J. Quinn and S. Rodriguez, Phys. Rev. 128, 2487 (1962).

with one of the specimens. The ratio of room temperature to residual resistance for this specimen was 1.4×10^3. The direction of sound propagation was 10° from the $(10\bar{1}0)$ axis in the $(11\bar{2}0)$ plane. The magnetic field vector was rotated in $(10\bar{1}0)$ and $(11\bar{2}0)$ planes, and varied continuously and automatically from 5 to 20 kG at each orientation where observations were made. A typical chart recording is shown in Fig. 2.

Several experiments were conducted on two zinc specimens. The purer specimen is characterized by a residual resistance ratio of 3×10^4. The direction of sound propagation for both specimens was within 5° of the (0001) axis. The magnetic-field vector was rotated in the $(10\bar{1}0)$ and $(11\bar{2}0)$ planes, and also about the (0001) axis. In the third case an angle of about 85° was preserved between **q** [and the (0001) axis] and **H** for all orientations. The magnetic field was varied continuously between 5 and 20 kG at each orientation.

EXPERIMENTAL RESULTS

Periodicity of the Oscillations. The periods observed agree with those observed for oscillations of the magnetic susceptibility at all orientations of the magnetic field. Although complicated beat patterns occur and the analysis is difficult, there is no significant departure from the constant periods observed in susceptibility measurements as the magnetic field is varied between 5 and 20 kG. The periods do not change as the sound frequency is varied from 27 to 81 Mc/sec.

In magnesium four sets of oscillations sinusoidal in character and periodic in $1/H$ were observed. Measurements were taken at several orientations of the magnetic field with ultrasonic frequencies of 27, 45, 63, and 81 Mc/sec, to determine any variation of the periods with the wavelength of the sound. The periods were found to vary with orientation of the magnetic field in the same way as those observed by Priestly[12] and by Gordon, Joseph, and Eck[13] in measurements of the magnetic susceptibility. The accuracy of our measurements is not quite as great as that of the magnetic susceptibility data; however, within the experimental uncertainties the periods are identical.

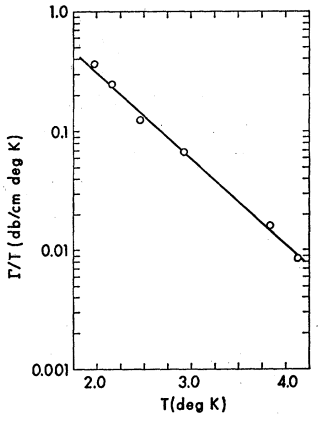

FIG. 3. Temperature dependence of the amplitude of a β-orbit oscillation in magnesium. **H** is in the $(11\bar{2}0)$ plane, 5° from the $(10\bar{1}0)$ axis.

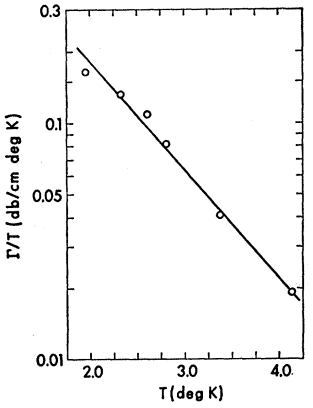

FIG. 4. Temperature dependence of the amplitude of an α or cigar orbit oscillation in zinc. **H** is in the $(11\bar{2}0)$ plane, 5° from the (0001) axis.

Three sets of oscillations periodic in $1/H$ were observed in zinc. Measurements were taken at two or three orientations of the magnetic field with ultrasonic frequencies of 27, 45, 63, and 81 Mc/sec to determine any variation of the oscillation periods with the wavelength of the sound. No such variation was observed. Variation of the periods with orientation of the magnetic field is the same as that observed by Joseph and Gordon[14] in measurements of the magnetic susceptibility.

Temperature Dependence of the Amplitude. The amplitude of oscillations in the acoustic attenuation of the magnesium and the less pure of the zinc specimens varies in the same way with temperature as de Haas–van Alphen oscillations, that is, Γ/T and χ/T vary approximately exponentially with T. Effective masses computed from the temperature dependence agree for the two phenomena within experimental uncertainty.

In magnesium the temperature dependence of the amplitude was measured at several orientations and in all cases Γ/T varies exponentially with T. In Fig. 3 this function is plotted for an α or cigar orbit oscillation with **H** in the $(11\bar{2}0)$ plane, 5° from the $(10\bar{1}0)$ axis. This exponential behaviour is characteristic of de Haas–van Alphen oscillations where many Landau levels participate in the absorption. The effective mass calculated from these data for the case shown in Fig. 3 is $0.20m$, in close agreement with the value $0.22m$ obtained by Priestley for the same orientation of the magnetic field and the same oscillation.

Oscillations having an amplitude of $\sim$0.5 dB/cm were observed with the less pure of the two zinc specimens and the temperature dependence of an A oscillation (α or cigar orbit), when the magnetic field vector is about 85° from the (0001) axis, is shown in Fig. 4. As might be expected, this lower amplitude oscillation displays a de Haas–van Alphen temperature dependence. A calculation of the effective mass yields the value $0.13m$, which is compatible with the value

[12] M. G. Priestley, Proc. Roy. Soc. (London) **A276**, 258 (1963).
[13] W. L. Gordon, A. S. Joseph, and T. G. Eck, *The Fermi Surface* (John Wiley & Sons, Inc., New York, 1960).
[14] A. S. Joseph and W. L. Gordon, Phys. Rev. **126**, 489 (1962).

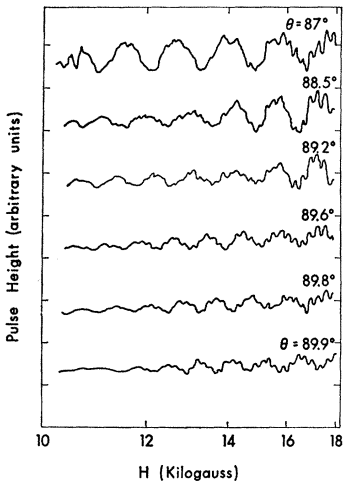

FIG. 5. Decrease in amplitude of oscillations in zinc as the angle θ between **q** and **H** approaches 90° **H** is in the $(10\bar{1}0)$ plane, and **q** is 3° from the (0001) axis. A similar decrease in amplitude occurs when θ approaches 90° from larger angles.

$0.10m$ obtained by Joseph and Gordon for the same orbit.

A few data obtained at 1.8 and 4.2°K with the purer zinc specimen indicate rather inconclusively that the amplitude of the oscillations may vary inversely with temperature, which is typical of giant quantum oscillations, but experimental difficulties interfered with further measurements on this specimen.

Line Shape of the Oscillations. The oscillations are sinusoidal in character, as are oscillations of the magnetic susceptibility.

Amplitude of the Oscillations. The maximum amplitude of the oscillations in the experiments with magnesium, realized at 81 Mc/sec and with a magnetic field of 20 kG, was 0.8 dB/cm. This compares with a zero-field attenuation at the same ultrasonic frequency of approximately 18 dB/cm. (Since only the first and second echo pulses were observed, and the second had very low amplitude, this figure is only accurate to ~50%.) The low amplitude of the oscillations suggests that they are characteristic of the de Haas–van Alphen regime where electrons from many Landau levels participate in the absorption, and where the periodicity is characteristic of an extremal area of the Fermi surface according to Eq. (3).

Difficulties arise in comparing oscillation amplitudes to the zero-field attenuation. Not only is the zero-field attenuation measured with very low accuracy, but a much greater limitation occurs because it is impossible to separate contributions to the zero-field attenuation arising from groups of electrons on different sheets of the Fermi surface. Thus comparison of the amplitude of a particular oscillation arising from a single sheet to the zero-field attenuation yields a ratio smaller than the one required for comparison with theory. However, Skobov's expression for this ratio is $(\hbar\omega_c/E_f)^{1/2}q_zl/\pi$ and if we equate this to the observed ratio for magnesium we find $q_zl\sim 1$. This is consistent with the residual resistivity of this specimen which yields $ql=1$ if m^* is assumed to be $\frac{1}{5}$ of a free-electron mass (m) in the expression $\sigma=ne^2\tau/m^*$ for the zero-field conductivity. The effective masses measured by Priestley vary from $0.01m$ to $1.0m$.

For the purer zinc specimen the maximum amplitude of the oscillations was approximately 5 dB/cm at a sound frequency of 81 Mc/sec and with a magnetic field near 20 kG. This compares with a zero-field attenuation at the same frequency of approximately 8 dB/cm. (As was observed for magnesium, this figure is accurate only to approximately 50%.) Thus the ratio of oscillation amplitude to zero-field attenuation is much greater than the ratio (0.8/18) with magnesium. Toxen and Tansal have observed an oscillation amplitude twice as great as the zero-field attenuation in bismuth, which may be compared with the $\frac{2}{3}$ ratio in zinc.

The value of ql for these measurements may be estimated, just as for magnesium, by using the residual resistivity of the specimen to estimate the electron mean free path. If the average effective mass in the conduction band is taken to be $0.2m$, a value that is central to the range of effective masses found by Joseph and Gordon, then $ql\sim 15$ with a sound frequency of 81 Mc/sec. Skobov's expression for the ratio of the oscillation amplitude to the zero-field attenuation, $(\hbar\omega_c/E_f)^{1/2}\times q_zl/\pi$, yields $q_zl\sim 15$ for these oscillations, in reasonable agreement with the foregoing estimate. As an example, oscillations corresponding to an orbit with an effective mass of $0.2m$ have an amplitude of 5 dB/cm at 20 kG.

The dependence of amplitude on magnetic field is difficult to measure due to the fact that usually oscillations from several Fermi surface segments form a complicated pattern. However, the over-all behavior in both magnesium and zinc for all orientations of the magnetic field is an oscillation amplitude proportional to the magnetic field between 5 and 20 kG. For the de Haas–van Alphen regime Skobov predicts amplitude proportional to $H^{1/2}$, and Quinn and Rodriguez predict $H^{3/2}$ dependence, either of which, it might be argued, could be consistent with these experiments.

Two difficulties occur in reconciling experimental results with theory. (a) The amplitude of the oscillations does not depend on the angle between the magnetic field and direction of sound propagation, as the Skobov and Liu and Toxen theories predict. The amplitude varies linearly with ql, or $\omega\tau$, but not with q_zl. Determination of the variation of amplitude with q_zl is difficult when **H** is rotated rather than **q**. However, despite factors that cause the amplitude to fluctuate when **H** is rotated, it is apparent that the oscillation amplitude is not proportional to $\cos\theta$ in either magnesium or zinc. In magnesium there is no variation and in zinc, where the ql value is higher, the amplitude of the oscillations remains approximately constant when θ is in the range 0 to 87°, and decreases in the range from 87° to 90°. An example of this decrease is shown in Fig. 5 (b). The ratio of oscillation amplitude to zero-

field attenuation is difficult to determine experimentally since, as mentioned previously, it is impossible to attribute an appropriate fraction of the zero-field attenuation to the group of electrons responsible for a particular oscillation. Thus the ratios which can be determined are invariably lower than the ones required for comparison with theory. However, the maximum amplitudes observed are the same order of magnitude as the Skovov prediction. The expression of Eq. (6) was derived assuming $ql \gg 1$; however, from the experimental results it appears that a very similar expression would be obtained if this condition were relaxed and $ql \sim 1$. Because of the limited resolution of the experimental technique at the present time, no oscillations can be observed when $ql \ll 1$.

The decrease in anplitude when θ is near 90° shown in Fig. 5 is less pronounced than the decrease that was observed in bismuth by Toxen and Tansal. They attempted to relate this effect to the fact that the group velocity required to satisfy $v_z^0 = v_s/\cos\theta$ will increase beyond the Fermi velocity v_f for large enough θ. This would only occur when $\cos\theta \sim 10^{-3}$, that is for $\theta \sim 89.9°$, whereas the decrease in amplitude is pronounced at angles much farther from 90° where, indeed, v_z^0 is less than $10^{-1}\, v_f$ unless v_f is anomalously low.

Although v_f may be much lower than the free-electron value on small segments of the Fermi surface it seems unlikely even in bismuth that sufficient electrons would be involved with low enough velocities ($\sim 10^{-2}$ of the free-electron value) to account for an oscillation observable with an anomalous amplitude over an angle of several degrees. In zinc all observed oscillations show the amplitude decrease over a 3° range so that small segments of the Fermi surface are certainly not sufficient to account for this behavior.

The argument that follows may offer a more plausible explanation of the amplitude variation with θ. The amplitude of the quantum oscillations depends strongly on the number of Landau cylinders participating in the acoustic absorption, and decreases rapidly as that number increases from one. The number participating will depend on $\omega\tau$ and the topology of the regions of the Fermi surface where $k_z = k_z^0$. According to Eq. (5) when $\omega\tau < 1$, a range of electron wave vectors satisfies the momentum and energy conservation conditions in the electron-phonon interaction. Only when θ approaches closely to 90° does the plane in k space defined by k_z^0 move from a region where the tangent to the Fermi surface is nearly parallel to the magnetic field, to a region where there is an appreciable angle between the two. Under these conditions an increased number of Landau levels will intersect the Fermi surface within the range of allowed k_z specified by Eq. (5) and the amplitude of the oscillations should then decrease. When $\theta = 87°$, the k_z^0 plane is approximately one fiftieth of the distance from the plane $k_z = 0$ to the plane of maximum Fermi momentum in the z direction. Thus there will be very little effect as θ is increased from 0 to 87° and possibly a considerable effect thereafter.

Effect of the electron spin. Spin-splitting of absorption peaks was observed in zinc for oscillations identified by Joseph and Gordon to correspond with the α or cigar orbit. The results obtained are the same as those reported and discussed briefly by Myers and Bosnell.[15] It should perhaps be emphasized that the spin-splitting of the absorption peaks remains a constant fraction of the period $\Delta(1/H)$ of the oscillations, for all orientations of the magnetic field. The g factors calculated by Myers and Bosnell from the spin-splitting vary with orientation of the magnetic field, but this variation is due entirely to variation of the effective mass, which is obtained from other sources and used in the calculation. No spin-splitting of absorption peaks was observed in magnesium.

SUMMARY

Low-amplitude quantum oscillations were observed in magnesium and zinc when $ql \sim 1$. These oscillations are similar to de Haas–van Alphen oscillations of the magnetic susceptibility in periodicity, temperature dependence of the amplitude, and line shape. The Skobov theory of acoustic attenuation for the de Haas–van Alphen regime is successful in predicting all three features. It is noted that the Skobov expression, Eq. (6), is identical in form with expressions derived for quantum oscillations of the magnetic susceptibility with the exception of a coefficient determining the amplitude.

The oscillations of much larger amplitude that have been observed in the purer zinc specimen are characteristic of a regime intermediate between the de Haas–van Alphen and giant-quantum-oscillations regimes. Difficulty exists here in comparing experimental results with theory since in this regime the acoustic attenuation has been expressed only in the form of an integral equation derived by Liu and Toxen. The amplitude of the oscillations depends on the fact that the sound wave can interact with electrons on only a few Landau cylinders whose number is determined by a rather narrow resonance factor. The number of Landau levels participating is approximately proportional to $1/\omega\tau$ and thus the amplitude is approximately proportional to $\omega\tau$.

The amplitude of all oscillations observed in zinc and magnesium is proportional to the sound frequency, but not to the angle between $\mathbf{q}$ and $\mathbf{H}$, except when this angle is near 90°. Thus the amplitude is proportional to $\omega\tau$ or $q_z l_z$ but not to $q_z l$. This behavior is not consistent with the conclusions stated by Liu and Toxen, although a reinterpretation of their calculations in which l is replaced by l_z appears reasonable.

[15] A. Myers and J. R. Bosnell, Phys. Letters **17**, 9 (1965).

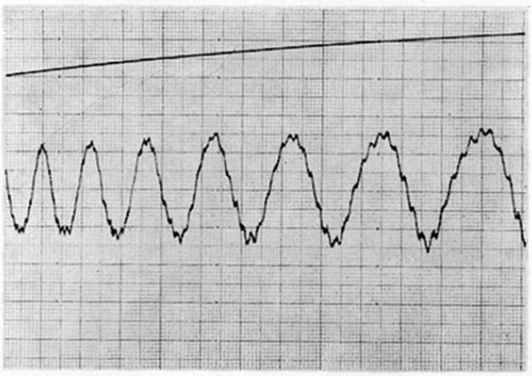

FIG. 2. Chart recording showing oscillations of the acoustic attenuation in magnesium. The magnetic field is in the $(11\bar{2}0)$ plane, 20° from the (0001) axis, and is approximately 20 kG when the smooth Hall-probe trace is full scale. (The oscillatory trace monitoring the pulse amplitude is not zeroed.)