In order to illustrate the implications of these remarks, we present in Fig. 2 a simple numerical comparison among the various approximations in third order. In constructing the figure, we have used the calculations of Baker, Gammel, and Hill,[14] based upon a finite square-well potential. These have been supplemented by R. Coldwell's calculations[15] of the anomalous terms for the same potential. The only parameter which appears is $k_F{}^0c$, where c is the radius of the well; the comparison is independent of the sign of the potential. The three Λ approximations contribute to ϵ_3 according to the weights given in Fig. 1; Brueckner theory corresponds to (b) plus the normal part of (a). The Iwamoto[12] extension of Brueckner theory to include hole-hole scattering is identical (in this order) with Λ_{11}. The physical region for the repulsive core is $k_F{}^0c \sim 1.4 \times 0.5 \approx 0.7$. The attractive part of the potential corresponds to a larger value of this parameter.

In the region $k_F{}^0c \lesssim 1$, it is clear from Fig. 2 that Λ_{10} is dramatically closer to perturbation theory than the other approximations discussed. Indeed, anomalous contributions *have* compensated for neglected normal terms.

We have made these comparisons in order to address ourselves to the question often asked by perturbation theorists, "What diagrams are included?" For reasons given above, we do not believe that this comparison is necessarily the best criterion. Furthermore, a theory (such as Brueckner theory) which sums an infinite subset of diagrams is no longer perturbation theory in the sense of being an expansion in powers of the interaction. Indeed, one common interpretation of such theories is that they are expansions in the number of particles simultaneously correlated. This is, in fact, the viewpoint propounded by Bethe.[13]

ACKNOWLEDGMENT

We wish to thank Professor D. S. Koltun for valuable discussion. We are indebted to R. Coldwell for his analysis and numerical calculation of the anomalous third-order terms.

[14] G. A. Baker, J. L. Gammel, and B. J. Hill, Phys. Rev. **132**, 1373 (1963).
[15] R. Coldwell (unpublished calculations).

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Numerical Calculation of the Ground-State Properties of Nuclear Matter Using the Λ_{10} Approximation*†

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This paper investigates numerically for the case of nuclear matter the consequences of one particular method of terminating the infinite chain of coupled Green's-function equations. Of special interest is a comparison of this Λ_{10} theory with the Λ_{00} theory investigated previously. These two approximate theories rely on solution of a T-matrix equation and they differ in the selection of the propagator for intermediate states. Using separable, nonlocal potentials the T-matrix equations are solved and the bulk properties of the system obtained. Results are presented for the energy and number densities, the effective mass, the nuclear compressibility, and the two-particle correlation function. It is found that the two theories give similar results, with Λ_{10} predicting several MeV less binding than Λ_{00} does.

I. INTRODUCTION

THE Martin-Schwinger[1] thermodynamic Green's-function approach to the many-fermion problem can be shown formally to give the exact ground-state properties of the system if the exact one-particle Green's function can be determined. This unfortunately is a difficult or impossible task with even semirealistic two-body potentials, and various approximations have been suggested. One class of these approximations involves solution of a T matrix equation and should be applicable to low-density systems with short-range interparticle forces. These approximate theories are analogous to Brueckner's theory,[2] which approached the problem from a perturbation-theory point of view. It is the purpose of this paper to investigate numerically one of these approximate Green's-function theories by means of a calculation of the ground-state properties of nuclear matter. Of particular interest is a comparison of this Λ_{10} theory with the Λ_{00} theory[3,4] previously investigated. Section II lists with very little derivation the basic

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[1] P. C. Martin and J. Schwinger, Phys. Rev. **115**, 1342 (1959).
[2] K. A. Brueckner and J. L. Gammel, Phys. Rev. **109**, 1023 (1958).
[3] R. D. Puff, Ann. Phys. (N. Y.) **13**, 317 (1961).
[4] D. S. Falk and L. Wilets, Phys. Rev. **124**, 1887 (1961).

equations needed to solve this problem and discusses the two-body interaction used in solving them. In Sec. III numerical results are presented for the binding energy, density, effective mass, compressibility, and two-particle correlation function. Section IV contains a discussion of these results.

II. THEORY

Thermodynamic Green's functions and their application to the many-fermion problem are discussed elsewhere in the literature and a repetition of this discussion will be omitted here. The reader is referred in particular to Martin and Schwinger[1] for a general treatment and to Puff[3] for an application to nuclear matter. Likewise a discussion of the Λ_{00} [3,4] and Λ_{10} [5] theories can be found and only an outline of their development will be presented here.

The bulk properties of the system can be obtained from the one-particle Green's function and it is convenient to get G_1 from the integral equation

$$G_1(11')=G_1^0(11')+iG_1^0(15)\langle 52|v|34\rangle G_2(34;2^+1'), \quad (1)$$

where the notation is that of the preceding paper. G_2 is obtained from a similar equation in terms of G_3:

$$G_2(12;1'2')=G_1^0(11')G_1(22')-G_1^0(12')G_1(21') +iG_1^0(13)\langle 34|v|56\rangle G_3(256;4^+1'2'). \quad (2)$$

For systems where the correlation length is less than or comparable to the interparticle spacing a reasonable approximation should be to factor the G_3 of Eq. (2) into properly symmetrized products of G_1 and G_2, keeping correlations between particles interacting through v. This assumes that while two particles in the medium are interacting their motion is essentially unaffected by the remaining particles in the medium. The approximation can be expressed analytically as

$$\langle 45|v|23\rangle G_3(123;1'2'3')\approx\langle 45|v|23\rangle[G_1(11')G_2(23;2'3') -G_1(12')G_2(23;1'3')+G_1(13')G_2(23;1'2')] \quad (3)$$

and using it the equation for G_2 becomes

$$G_2(12;1'2')\approx G_1(11')G_1(22')-G_1(12')G_1(21') +\tfrac{1}{2}i[G_1^0(13)G_1(24)+G_1(13)G_1^0(24)] \times\langle 34|v|56\rangle G_2(56;1'2'). \quad (4)$$

The quantity $\frac{1}{2}[G_1^0G_1+G_1G_1^0]$ is defined as $-i\Lambda_{10}(1234)$ and Λ will turn out to be the propagator in a T matrix equation. Puff, in first applying the Green's-function methods to this problem, made the additional approximation of replacing the one-particle Green's functions in this propagator by free-particle G_1^0's. This results in what is called the Λ_{00} approximation. It seems that the Λ_{10} theory is a more "natural" approximation to the exact theory, but at least for certain classes of potentials Λ_{00} is easier to treat numerically. A third approximation is also possible, the Λ_{11} approximation, which results when the propagator consists of a product of two G_1's. The Λ_{11} theory can be shown to have certain symmetry properties and to obey certain conservation conditions that the other two Λ theories violate.[6]

Equation (4) can be transformed to momentum-energy space and converted to an integral T matrix equation[5]:

$$\langle \mathbf{k}_1\mathbf{k}_2|T(z)|\mathbf{k}_1'\mathbf{k}_2'\rangle=\langle \mathbf{k}_1\mathbf{k}_2|v-v_{\rm ex}|\mathbf{k}_1'\mathbf{k}_2'\rangle +\int_{k_3k_4}\langle \mathbf{k}_1\mathbf{k}_2|v|\mathbf{k}_3\mathbf{k}_4\rangle\Lambda(\mathbf{k}_3\mathbf{k}_4z)\langle \mathbf{k}_3\mathbf{k}_4|T(z)|\mathbf{k}_1'\mathbf{k}_2'\rangle, \quad (5)$$

where Λ differs depending on the particular approximation employed. For Λ_{00}, in the zero-temperature limit and with the condition that the chemical potential μ be negative, the propagator is[3]

$$\Lambda_{00}(\mathbf{k}_1\mathbf{k}_2\omega)=[\omega-k_1^2-k_2^2+2\mu]^{-1}. \quad (6)$$

Here the difference from Brueckner theory[2] is apparent: the intermediate-state energies are simply free-particle energies and there is no exclusion from scattering into the Fermi sea. The Brueckner propagator includes a projection operator which requires that the intermediate states be outside the Fermi sea and the Brueckner energy denominator contains two "dressed" particles in the intermediate state.

Similarly for negative μ and zero temperature the Λ_{10} propagator can be shown to be[5]

$$\Lambda_{10}(\mathbf{k}_1\mathbf{k}_2\omega)=\frac{1}{2}\left[\frac{1}{\omega-k_2^2+\mu-k_1^2+\mu-V(k_1,\omega-k_2^2+\mu)} -\frac{\rho(k_1)\theta(-\omega_0(k_1))}{\omega-k_2^2+\mu-\omega_0(k_1)}\right]+(k_1\leftrightarrow k_2), \quad (7)$$

where V, ω_0, and ρ are defined below. Inside the Fermi sea the two terms in brackets tend to cancel and so the Pauli exclusion principle is partially incorporated in Λ_{10}. Also the energy denominator contains one free and one "dressed" particle, not two free particles as in the Λ_{00} case.

Equation (5) can be solved with the two above propagators and the ground-state properties are obtained

[5] R. Puff, A. S. Reiner, and L. Wilets, preceding paper, Phys. Rev. 147, 778 (1966).

[6] G. Baym and L. P. Kadanoff, Phys. Rev. 124, 287 (1961).

through:

$$V(k_1,\omega)=\int_{\mathbf{k}_2}\rho(k_2)\langle\mathbf{k}_1\mathbf{k}_2|\,T(\omega+\omega_0(k_2))\,|\mathbf{k}_1\mathbf{k}_2\rangle\theta(k_F-k_2)\,,$$

$$\omega_0(k_1)=k_1^2+V(k_1,\omega_0(k_1))-\mu\,,$$

$$\rho(k_1)=\left[1-\frac{\partial V(k_1,\omega)}{\partial\omega}\bigg|_{\omega=\omega_0(k_1)}\right]^{-1},$$

$$\rho=\int_{\mathbf{k}_1}\rho(k_1)\theta(k_F-k_1)\,,$$

$$E/N=\rho^{-1}\int_{\mathbf{k}_1}\rho(k_1)\tfrac{1}{2}[\omega_0(k_1)+k_1^2+\mu]\theta(k_F-k_1)\,,$$

$$r_0=[3/4\pi\rho]^{1/3},$$

$$\mu=k_F^2+V\quad(k_F,\omega=0)\,.\tag{8}$$

A self-consistent procedure must be used to solve the first three of the above equations.

Equation (5) for the T matrix is very difficult to solve with realistic nuclear potentials. But since one of the main purposes of this work is to compare numerically the Λ_{00} and Λ_{10} theories, it should be possible to employ a potential which retains at least the general features of nuclear matter but which is easier to work with numerically than the complete potential. Two particular potentials will be used, both of which have the form of the sum of two separable, nonlocal potentials. Their advantage is obvious, as it will be shown below that with them a closed form expression for $T(\omega)$ can be obtained. However, it should be realized that in doing this a comparison of the end results of the calculation with experiment may not be meaningful.

The two particular forms of this semirealistic potential are the Puff potential,[3]

$$\langle k|v|k'\rangle=(2\pi)^{-3}2\lim_{\lambda_c\to\infty}\lambda_c\frac{\sin kr_c}{k}\frac{\sin k'r_c}{k'}-2\lambda\frac{1}{k^2+\beta^2}\frac{1}{k'^2+\beta^2}\,,\tag{9}$$

and the back-to-back Yamaguchi potential,[7]

$$\langle k|v|k'\rangle=(2\pi)^{-3}2\lambda_1\frac{1}{k^2+\beta_1^2}\frac{1}{k'^2+\beta_1^2}-2\lambda_2\frac{1}{k^2+\beta_2^2}\frac{1}{k'^2+\beta_2^2}\,.\tag{10}$$

k and k' are the relative momenta $\frac{1}{2}|\mathbf{k}_1-\mathbf{k}_2|$ and $\frac{1}{2}|\mathbf{k}_1'-\mathbf{k}_2'|$, respectively. These two potentials have no tensor part and interact in relative s states only. Their parameters differ depending on whether the interaction is in the singlet or triplet state, and all the parameters must be fit to empirical scattering data. The Λ_{00} T matrix equation can be solved exactly with each of the above potentials, but the numerical methods that must be used to obtain $T(\omega)$ for the Λ_{10} theory make the Yamaguchi potential preferable there.

Most of the calculations reported here were performed with the back-to-back Yamaguchi potential of Eq. (10). Using it the solution to the direct part of Eq. (5) can be shown to be

$$\langle\mathbf{k}_1\mathbf{k}_2|\,T(\omega)\,|\mathbf{k}_1'\mathbf{k}_2'\rangle=\langle k|\,T_K(\omega)\,|k'\rangle$$
$$=2(2\pi)^3\frac{\lambda_1(1+\lambda_2L_{22})v_1(k)v_1(k')-\lambda_1\lambda_2L_{12}[v_1(k)v_2(k')+v_1(k')v_2(k)]-(1-\lambda_1L_{11})\lambda_2v_2(k)v_2(k')}{(1-\lambda_1L_{11})(1+\lambda_2L_{22})+\lambda_1\lambda_2L_{12}^2}\,,\tag{11}$$

where

$$v_i(k)=(k^2+\beta_i^2)^{-1}$$

and

$$L_{ij}=2\int d^3k\;v_i(k)v_j(k)\Lambda(k_1k_2\omega)\,.$$

With the Λ_{00} propagator the L_{ij} can be obtained exactly, but for Λ_{10} numerical integration techniques must be used and the method of Gaussian quadratures was employed. Equation (11) is valid for the Puff potential case also, if $v_1(k)=(\sin kr_c)/k$ and the $\lambda_1\to\infty$ limit is taken.

Puff[3] fitted the λ and β of Eq. (9) to the low-energy scattering length and effective range and chose a physically reasonable value for the shell radius. His values for these parameters are listed in Table I. Using them it is possible to calculate high-energy phase shifts, which can be used to determine the parameters of the Yamaguchi potential. They too are found in Table I. Subsequent to Puff's original calculations better empirical determinations of the high-energy phase shifts were made and the Yamaguchi parameters were fit to them. In the

TABLE I. Potential parameters fitted to Puff scattering data.

	Puff potential			Yamaguchi potential	
	Singlet	Triplet		Singlet	Triplet
β (F^{-1})	2.004	2.453	β_1 (F^{-1})	3.0	3.5
λ (F^{-3})	3.640	8.695	β_2 (F^{-1})	2.4162	2.9922
r_c (F)	0.45	0.45	λ_1 (F^{-3})	87.908	557.81
cotδ	−2.4227	−9.2313	λ_2 (F^{-3})	33.979	212.55
E (MeV)	310	310	cotδ	−2.4227	−9.2313
			E (MeV)	310	310

[7] Potentials of this type were first used by R. C. Kennedy. See R. C. Kennedy, doctoral dissertation, University of Washington, 1964 (unpublished), and also R. C. Kennedy, L. Wilets, and E. M. Henley, Phys. Rev. 133, B1131 (1964).

TABLE II. Yamaguchi potential parameters fitted to "best" scattering data.

		β_1 (F^{-1})	β_2 (F^{-1})	λ_1 (F^{-3})	λ_2 (F^{-3})	cotδ	E (MeV)
A	singlet	3.0	1.7654	7.6393	1.7269	−6.3713	310
B	triplet	4.5	2.1190	54.846	4.4578	24.347	300
C	triplet	3.5	2.2379	25.763	6.8618	24.347	300
D	triplet	5.5	2.0798	186.80	4.0928	24.347	300
E	singlet	4.5	1.6451	25.825	1.0230	−6.3713	310
F	singlet	6.0	1.6199	122.45	0.95083	−6.3713	310
G	triplet	4.5	2.0740	45.325	3.7858	3.9442	220
H	triplet	4.5	2.0116	35.048	3.0273	17.067	140

singlet case the phase shift of MacGregor, Moravcsik, and Stapp[8] at 310 MeV was used, while in the triplet case the Gammel-Christian-Thaler[9] phase shifts were used. These latter theoretical values agree with the phase shifts of Hull, Lassila, Ruppel, McDonald, and Breit.[10] Table II summarizes the various sets of these "best" parameters, some chosen to investigate the dependence on the free parameter β_1 and some to investigate the high-energy point at which the phase shift is fitted. There is no fundamental reason to select one member of these families over the others and no attempt was made to do so. The complete Λ_{00} and Λ_{10} calculations were made using sets of parameters *A* and *B* from Table II.

III. NUMERICAL RESULTS

It is desirable to obtain a self-consistent calculation of the binding energy over a range of density values, and this is accomplished by treating the Fermi momentum as a parameter and varying it. Of particular interest is the k_F value that gives the minimum in the energy versus density curve and that which yields the experimental density. This was first done using the Puff potential and the parameters of Table I in a Λ_{00} calculation. The results are in good agreement with those of Puff[3] and of Falk and Wilets.[4] Then Λ_{00} was investigated with the back-to-back Yamaguchi potential which fit the same scattering data as the Puff potential (Table I). The values of E/N and μ from these two calculations agree to within about 2 MeV in the density region of interest and would give better agreement if the free parameter β_1 were varied. Hence the bulk properties of the system do not depend significantly on whether the Puff or Yamaguchi potential is used. Finally Λ_{10} was solved with the Yamaguchi potential (Puff parameters). The results of these three calculations are found in Table III and the Yamaguchi potential results are graphically displayed in Fig. 1.

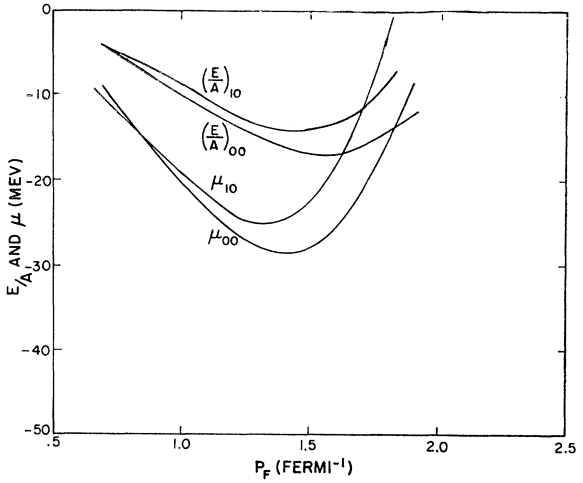

FIG. 1. Binding energy per particle and chemical potential calculated with the Yamaguchi potential (Puff parameters).

Calculations were also made for the Λ_{00} and Λ_{10} approximations using the Yamaguchi potential and the

TABLE III. Λ_{00} and Λ_{10} Puff parameter self-consistent results.

	Λ_{00} Puff potential			Λ_{00} Yamaguchi potential			Λ_{10} Yamaguchi potential		
k_F (F^{-1})	μ (MeV)	r_0 (F)	E/N (MeV)	μ (MeV)	r_0 (F)	E/N (MeV)	μ (MeV)	r_0 (F)	E/N (MeV)
0.7				−9.7	2.30	−4.1	−10.5	2.32	−4.4
0.9				−16.5	1.77	−7.5	−16.1	1.77	−7.2
1.1				−23.3	1.44	−11.4	−21.3	1.43	−10.3
1.2	−26.5	1.32	−13.5	−26.1	1.32	−13.3			
1.3	−28.6	1.22	−15.2	−27.9	1.22	−14.9			
1.325							−25.1	1.18	−13.6
1.4	−29.6	1.13	−16.5	−28.7	1.13	−16.2			
1.45							−23.7	1.08	−14.0
1.5	−29.2	1.06	−17.4	−27.9	1.06	−16.9			
1.575							−19.4	1.00	−13.4
1.6	−27.2	1.00	−17.6	−25.6	1.00	−16.9			
1.7							−11.6	0.93	−11.3
1.765	−20.1	0.91	−16.2	−17.8	0.91	−15.1			
1.82							+0.1	0.87	−7.2
1.88422	−11.8	0.86	−13.7	−9.	0.86	−12.3			

[8] M. H. MacGregor, M. J. Moravcsik, and H. P. Stapp, Ann. Rev. Nucl. Sci. **10**, 291 (1960).
[9] J. L. Gammel, R. S. Christian, and R. M. Thaler, Phys. Rev. **105**, 311 (1957).
[10] M. H. Hull, Jr., K. E. Lassila, H. M. Ruppel, F. A. McDonald, and G. Breit, Phys. Rev. **122**, 1606 (1961).

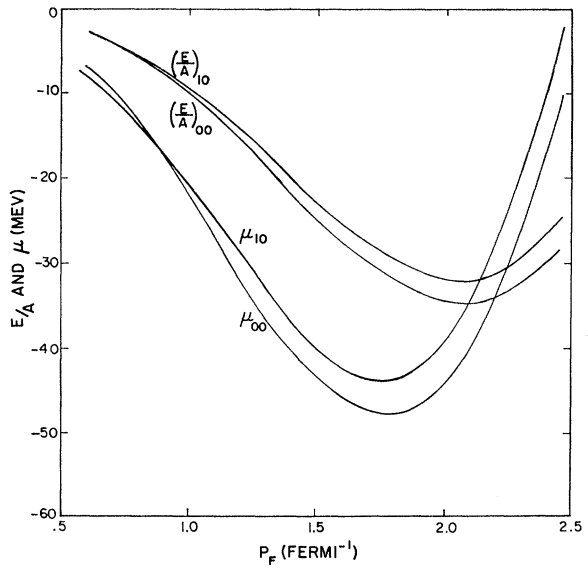

FIG. 2. Binding energy per particle and chemical potential calculated with the Yamaguchi potential (best parameters).

first two sets of "best" parameters from Table II. The results are presented numerically in Table IV and graphically in Fig. 2. Finally calculations were made with the other sets of parameters from Table II in order to investigate the choice of the free parameter and the dependence on the high-energy point at which the phase shift is fitted. The calculations were made at $k_F=1.525$ F^{-1}, corresponding to the empirical density, and at $k_F=2.025$ F^{-1}, corresponding to the energy minimum, and use the Λ_{00} approximation. The results are enumerated in Table V.

It is seen that the two approximate theories give qualitatively similar results for the cases investigated. In both cases Λ_{00} was more tightly bound than Λ_{10} by 3 MeV or less. The E/N-versus-k_F curves were fitted by the method of least squares to a third-order polynomial and the energy minima were found by differentiation. These results are found in Table VI.

TABLE IV. Λ_{00} and Λ_{10} Yamaguchi-potential best-parameter self-consistent results.

	Λ_{00}			Λ_{10}		
k_F (F^{-1})	μ (MeV)	r_0 (F)	E/N (MeV)	μ (MeV)	r_0 (F)	E/N (MeV)
0.6	−6.9	2.72	−2.8	−8.	2.7	−3.
0.85				−15.4	1.88	−6.8
1.00	−21.8	1.58	−10.3			
1.15				−27.2	1.36	−13.4
1.35				−35.2	1.15	−18.7
1.45	−41.7	1.07	−23.3			
1.575	−45.3	0.99	−26.8	−42.0	0.98	−24.8
1.80	−47.9	0.86	−31.9	−43.8	0.85	−29.5
2.025	−43.5	0.77	−34.4	−38.5	0.76	−31.8
2.25	−30.7	0.69	−33.4	−24.1	0.68	−30.2
2.35				−14.5	0.65	−28.1
2.45	−11.5	0.63	−28.7	−2.8	0.63	−25.

TABLE V. Λ_{00} Yamaguchi-potential best-parameter self-consistent results.

Singlet parameters (see Table II)	Triplet parameters (see Table II)	μ (MeV)	r_0 (F)	E/N (MeV)
$k_F=2.025$ F^{-1}				
A	B	−43.5	0.77	−34.4
A	C	−46.1	0.765	−36.0
A	D	−39.4	0.77	−32.0
E	B	−41.8	0.77	−33.4
F	B	−38.8	0.77	−31.6
A	G	−45.1	0.77	−35.2
A	H	−46.8	0.77	−35.9
$k_F=1.525$ F^{-1}				
A	B	−44.0	1.02	−25.4
A	C	−44.9	1.02	−26.0
A	D	−42.6	1.02	−24.7
E	B	−43.5	1.02	−25.1
F	B	−42.5	1.02	−24.6
A	G	−44.3	1.02	−25.6
A	H	−44.5	1.02	−25.6

With very little extra effort it is possible to calculate the nuclear compressibility and the effective mass at the Fermi surface. The usual definitions of these quantities are

$$K=r_0^2\partial^2(E/N)/\partial r_0^2,$$

$$\frac{m}{m^*}=\frac{1}{2k_F}\frac{d\omega_0(k)}{dk}\bigg|_{k=k_F}. \tag{12}$$

Calculations have been made for the four cases of interest and the results at the energy minima are presented in Table VI.

The two-body correlation function is of interest because it gives information on the range of correlations in the system. If the interparticle spacing is large compared to the correlation length an approximation such as Eq. (3) should be valid. If it is small a way must be found to retain the higher order correlations that are omitted by Eq. (3). $f(r)$, the two-body correlation function, gives the probability of finding two particles a distance r apart in the medium and can be calculated directly from G_2:

$$f(r)=f(|\mathbf{r}_1-\mathbf{r}_2|)=-G_2(r_1t,r_2t;r_1t^+,r_2t^+). \tag{13}$$

Reynolds and Puff[11] have shown that this G_2 can be calculated from the nondiagonal T matrix and their

TABLE VI. Energy minima, compressibility, and effective mass.

	E/N (MeV)	r_0 (F)	K (MeV)	m^*/m
Λ_{00}–Puff parameters	−17.0	1.03	200	0.60
Λ_{10}–Puff parameters	−14.1	1.08	210	0.57
Λ_{00}–best parameters	−34.5	0.75	440	0.49
Λ_{10}–best parameters	−31.8	0.75	450	0.49

[11] J. C. Reynolds and R. D. Puff, Phys. Rev. **130**, 1877 (1963).

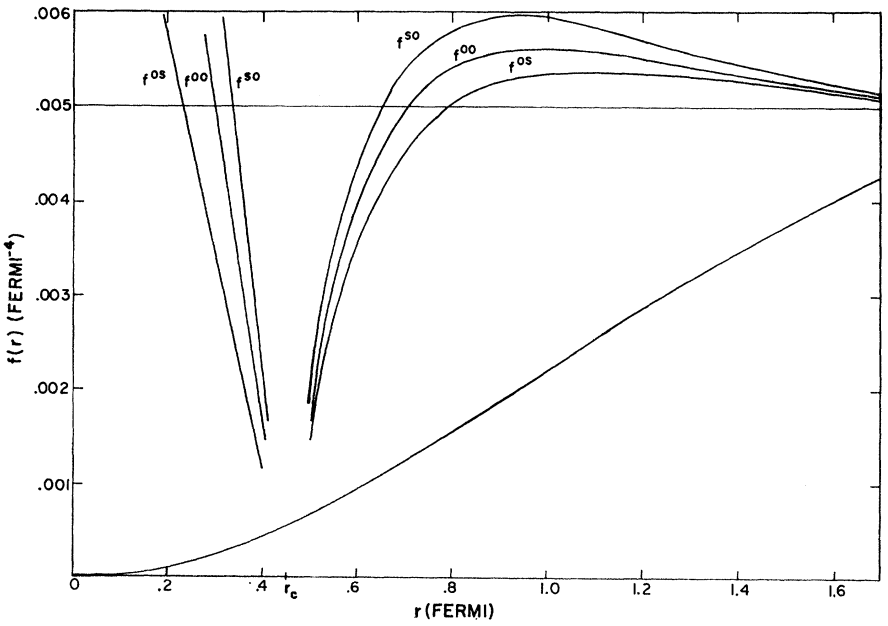

FIG. 3. Correlation function calculated with the Puff potential. The superscripts refer to cases where the spin and isotopic spin are same-same, same-opposite, opposite-same, and opposite-opposite, respectively.

argument will not be presented here. Calculations were made using the Λ_{00} approximation at $k_F=1.6865$ F^{-1} for the several potentials and sets of parameters and the results are graphically displayed in Figs. 3, 4, and 5. The results for the Puff potential agree with those of Reynolds and Puff.

IV. CONCLUSION

The results of the previous section are in agreement with the prediction of Puff, who stated that Λ_{10} should be slightly less tightly bound than Λ_{00}; he estimated the difference to be less than 15%.

One might notice that the calculated binding energy and density obtained using the Puff set of parameters agree fairly well with the experimental values, while the later "best" parameters predict considerably too much attraction. This should be considered neither a justification nor a condemnation of the Green's-function approach to the many-fermion problem, but rather should be regarded as a consequence of the two-particle potential used in obtaining the results. The potentials used here are sufficient to investigate the difference between the Λ_{00} and Λ_{10} theories but they cannot be expected to yield a result good enough to be compared

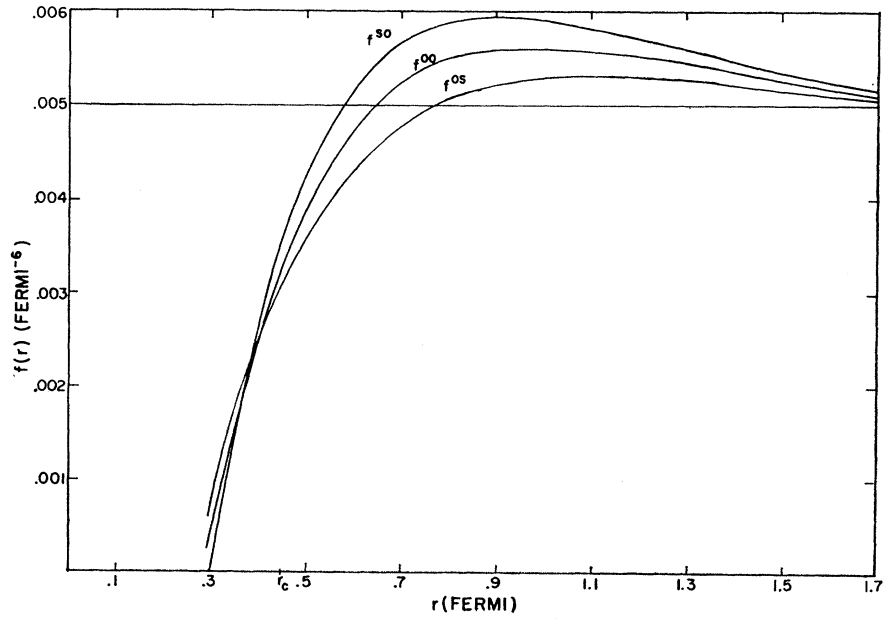

FIG. 4. Correlation function calculated with the Yamaguchi potential (Puff parameters).

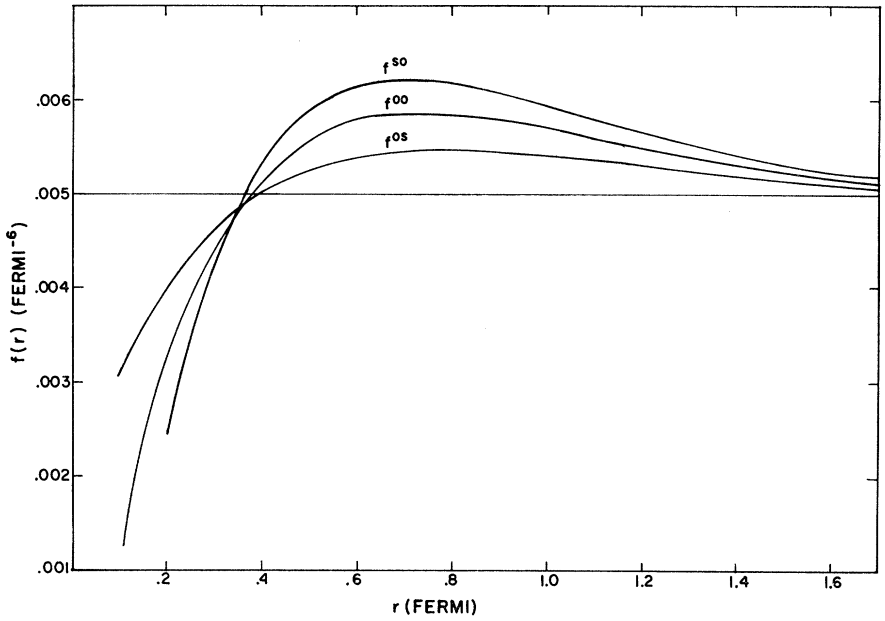

FIG. 5. Correlation function calculated with the Yamaguchi potential (best parameters).

with experiment. The Puff and Yamaguchi potentials are purely central in nature and occur only for relative s states. In reality d waves are important and p waves not negligible. Also Brueckner and Masterson[12] (using various Gammel-Thaler potentials) found that the central-tensor force mixture significantly affects the binding energy. A valid comparison with experiment must await calculations using more realistic potentials.

From Figs. 1 and 2 it is seen that neither the Λ_{00} nor the Λ_{10} approximation is thermodynamically consistent. For zero pressure the minimum value of E/N should equal the value of the chemical potential at the same density, while in both cases discussed here the crossing point of the E/N and μ curves occurs at too large a density. This is a consequence of the symmetry properties that were violated in making the approximations of Sec. II.

It should also be pointed out that the back-to-back Yamaguchi potential used here is a "soft" core potential and Wong[13] has shown that soft cores can produce 4–7 MeV more binding than truly hard cores.

Finally the calculation of the correlation function shows that the correlation distance in nuclear matter is of the same order of magnitude as the interparticle spacing. This indicates that approximating G_3 as was done in Sec. II is not a bad approximation for nuclear matter, and it agrees with the results of Bethe[14] and of Moszkowski[15] who found that three-body correlations, properly treated, do not significantly contribute to the energy of the system.

Note added in proof. Potentials of the Puff type have been applied to nuclear matter by DeDominicis[16] in a perturbation-theory calculation. He found that the binding energy per particle at densities greater than the normal density was significantly reduced when the triplet-state potential included a tensor part.

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