which is the commonly given relation[7] between pressure, Debye thermal energy, and Grüneisen "constant" γ. (Note that our pressure differs from the usual pressure by a convective term and that the volume average of the term linear in the displacement gradients vanishes.) Using (4.9), (4.8), and (4.6), we have calculated γ on a computer and find good agreement[2] with the values calculated for Si and Ge from[7]

$$\gamma=\alpha K/C_V\,, \tag{4.11}$$

connecting the volume expansion coefficient α, the bulk modulus K, and the specific heat per unit volume C_V.

[7] C. Kittel, *Introduction to Solid State Physics* (John Wiley & Sons, Inc., New York, 1956), 2nd ed., p. 154.

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Active CdS Ultrasonic Oscillator

D. L. White
Bell Telephone Laboratories, Murray Hill, New Jersey

AND

Wen-Chung Wang*
Syracuse University, Syracuse, New York

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An ultrasonic amplifier driven under dc conditions has been observed to oscillate in a stable high-frequency mode. The oscillation corresponded to a very high overtone of the fundamental acoustic resonance of the CdS plate, such as the 100th harmonic or greater. This oscillation is distinctly different from the periodic oscillations in current which occur in high-gain ultrasonic amplifiers at a period corresponding to an electron transit time, and is due to a buildup of acoustic flux. This new oscillation has very pure periodic waves which contain either one frequency or several harmonically related frequencies. Oscillations were detected by the rf current in the applied dc, or by ultrasonic transducers. There is no broad background of incoherent noise—all the available power goes into this one mode of oscillation. To oscillate, the CdS must have net round-trip ultrasonic gain, but not too much gain. Excessive gain allows a buildup of incoherent acoustic flux which destroys the high-frequency mode. The frequency of oscillation is near the frequency of the greatest ultrasonic gain. Although the exact mode on which the oscillator would resonate could not be predetermined, it could always be made to oscillate on the same mode under identical settings of voltage and resistivity.

When sound is amplified in cadmium sulfide[1] so that it has net round-trip gain, i.e., the gain in the direction of the electron current exceeds the loss in the reverse direction, one expects the plate to vibrate at its natural resonant frequencies.[2,3]

Such stable, high-frequency, cw oscillation has been observed in a cadmium sulfide plate in which ultrasonic energy is being amplified. The oscillations correspond to the cadmium sulfide plate vibrating on a very high overtone of the fundamental thickness mode of the plate, typically the 100th or even a higher harmonic. The surprising observation reported in this paper is the degree to which vibrational energy could be confined to one frequency and its harmonics.

The crystals in which these oscillations were observed were samples of pure cadmium sulfide which had been heated in a sulfur atmosphere at 1000°C for about 24 h.[4] The close agreement between the ultrasonic amplification characteristics and the small-signal theory observed in such material indicates that it is essentially trap-free. Two opposite faces of the crystal along the hexagonal axis were polished optically flat and parallel. They were diffused with a very thin indium layer to provide electrical contacts. Batteries were used as dc source. The oscillations were detected by monitoring the rf component of the current passing through the crystal.[5]

When the voltage is first turned on, a large number of frequencies grows in amplitude because they have round-trip gain. After a period of mode competition, lasting typically 1 msec, one mode dominates and sta-

* On leave from Syracuse University, Syracuse, New York.

[1] A. R. Hutson, J. H. McFee, and D. L. White, Phys. Rev. Letters **7**, 237 (1961); D. L. White, J. Appl. Phys. **33**, 2547 (1962).

[2] V. L. Gurevich and B. D. Laikhtman, Fiz. Tverd. Tela **6**, 2884 [English transl.: Soviet Phys.—Solid State **6**, 2299 (1965). E. C. Adler and G. W. Farnell, Proc. IEEE **53**, 2183 (1965).

[3] Papers by A. R. Hutson, E. C. Adler, D. L. White, and W. C. Wang presented at the International Ultrasonics Symposium, Boston, Massachusetts, 1965 (unpublished).

[4] D. L. White, E. T. Handelman, and J. T. Hanlon, Proc. IEEE **53**, 2157 (1965).

[5] The oscillations were also detected as sound by the use of quartz and evaporated-layer ultrasonic transducer attached to the sample. However, in this case the stress-free boundary conditions were no longer valid.

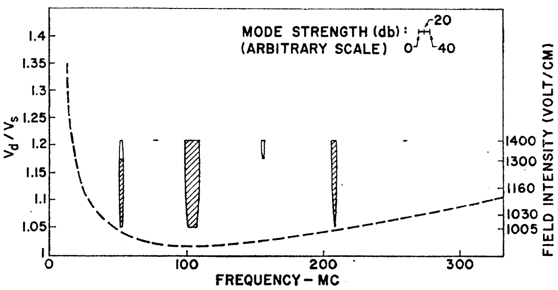

FIG. 1

FIG. 1. Mode pattern of 5.95×10^{-5} (Ω cm)$^{-1}$ crystal as a function of electron drift velocity (normalized by sound velocity). The field intensity is also shown in the figure. The width of the line is chosen to represent the mode strength.

bilizes. Under identical settings of voltages and resistivity, the crystal can be made to oscillate on the same mode. If there is only a small band of frequencies having net round-trip gain, the dominant frequency in the final mode is near the frequency of maximum gain.

Figures 1 and 2 show the typically mode spectrum as a function of voltage for two different resistivities of one crystal. Conductivity is controlled by illumination. The crystal thickness of 0.214 cm corresponds to a fundamental resonant frequency of 410 kc/sec, and the harmonic number of the fundamental is indicated as well as the frequency. Since the cadmium sulfide is non-Ohmic when the electron drift velocity exceeds the velocity of sound, the ordinate is given as a linear plot of current on the left (normalized as the ratio of drift velocity to sound velocity) and as the electric field on the right. The width of each line shows the strength of that frequency, the shaded parts indicating a single frequency with no immediate neighbors, and the unshaded portion at the top meaning that adjacent frequencies have appeared. The width of the line shows power output in a log scale (not frequency broadening).

When the crystal has a conductivity of 6×10^{-5} (Ω cm)$^{-1}$, as in Fig. 1, most of the energy is confined to a single frequency at 102 Mc/sec which is near the frequency of maximum gain. A harmonic, 31 dB down,

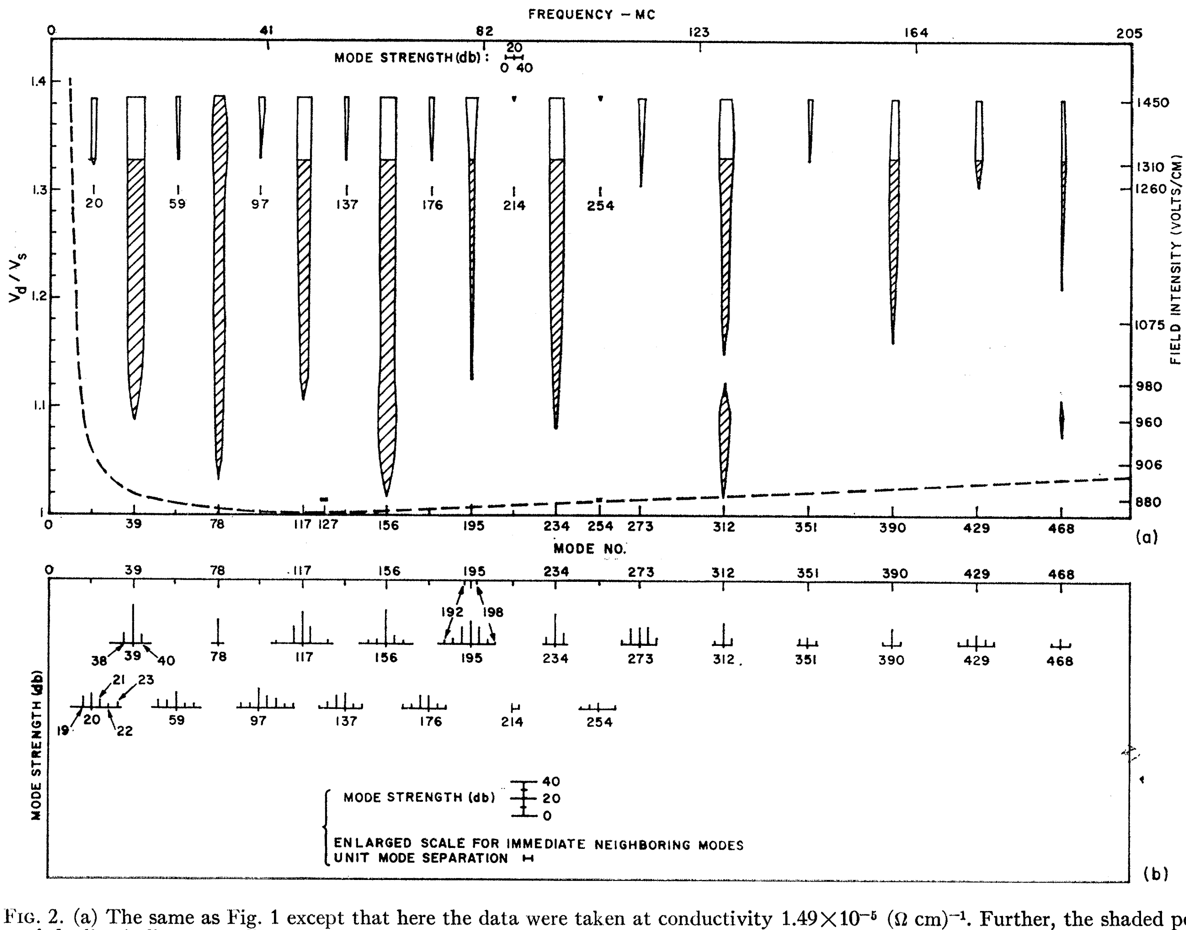

FIG. 2. (a) The same as Fig. 1 except that here the data were taken at conductivity 1.49×10^{-5} (Ω cm)$^{-1}$. Further, the shaded portion of the line indicates a solitary mode with no immediate neighbors. The unshaded part of the line indicates that the mode is the strongest of a group and some of its neighboring modes do exist but are not shown [see Fig. 2(b)]. (b) The mode pattern at $v_d/v_s=1.385$. Here, the height of a line expresses the mode strength. The mode number designated corresponds to the strongest mode in its group. It is noted that the separation among the strong modes (numbered) is represented by the same scale as in (a).

and a subharmonic, 30 dB down, are also generated but are small, most likely because they do not have net round-trip gain.

In more conductive materials there is a gain over a wider range of frequencies, complicating the mode spectrum. In the case of the sample in Fig. 2(a) where $\sigma = 1.5\times10^{-5}$ $(\Omega\ \text{cm})^{-1}$, a current of $v_d/v_s = 1.018$ produced oscillation on the 127th mode, but a higher current caused a shift to the 156th mode, and its harmonic—the 312th. As the current increases, the subharmonic (the 78th mode) appears which in turn generates its subharmonic, the 39th mode, at a higher current. Also under these conditions the harmonics of the 39th mode (or really the results of mixing between the dominant modes) are present. At still higher voltages the 39th mode cannot generate a subharmonic because it is an odd mode, and several modes, e.g., the 19th, 20th etc. are generated. These modes shown in Fig. 2(b) mix with the higher frequencies and each other to produce a complicated set of modes which no longer bear a simple relation to the dominant 156th.

The more conductive samples tend to oscillate primarily at one frequency, near the frequency of maximum gain, which is proportional to the square root of the conductivity. If a thick sample, such as the 2-mm crystal used in Figs. 1 and 2, is made too conductive, it oscillates near the fundamental frequency. This low-frequency instability suppressed the high-frequency overtone oscillations and only wide-band incoherent noise is observable at high frequencies. In order to oscillate on a pure-high-overtone mode, the crystal has to be thin enough to have less than approximately 20-dB gain at the frequency of maximum gain. In one crystal which was 0.1 mm thick, coherent oscillations have been observed up to 900 Mc/sec.

Since the frequencies are multiples of the fundamental acoustic resonance, the resonator is primarily acoustic in nature, but the amplitude of the waves is so large that nonlinear effects dominate the details of the oscillations. Due to the drift current, the velocity of sound in one direction differs from the velocity in the opposite direction, so standing waves are not formed.[3] A necessary condition of resonance is that the total phase change of the wave in the forward and backward directions must be an integral multiple of 2π.

According to small-signal theory at least one of the frequencies has a net round-trip gain, but for a stable oscillation, the amplitude must be so large that nonlinear effects reduce the round-trip gain to unity. The nonlinearity is due to the alternating electric fields accompanying the sonic waves using all available electrons in the conduction band to form the bunched carriers. Since the nonlinearity couples all sonic frequencies together, the separate frequencies observed are not normal modes as small-signal theory would predict, but are really the Fourier components of a single mode consisting of traveling wave packets. Linear theory predicts dispersion, but the observed stable wave forms show the frequency components are locked together and travelling at the same velocity.

The harmonically related frequencies form a stable mode, and other frequencies are suppressed. The greater stability of a set of harmonic frequencies is perhaps the explanation of the shift of the 127th acoustic harmonic to the 156th in Fig. 2. Since the 127th is odd, it could not generate a subharmonic and was not the most stable mode under higher drive.

The amplitude of the sonic wave must everywhere in the crystal be large enough to produce substantial nonlinear effects. If the amplitude on the negative side is too small, not only would the frequency components lose their phase coherence but the broad background of noise would no longer be suppressed. This could produce an unstable buildup in the incoherent (thermal) phonon background, resulting in the low-frequency oscillations mentioned above.

Since most of the energy of the waves is elastic, the frequency stability of the oscillator is comparable to crystal resonators. The change in frequency due to the variation in the applied dc voltage, resistivity, and temperature was observed to be correlated with the change in sound velocity as given in the small-signal theory. Frequency was stable to several parts per million and the line width considerably less than a part per million.