Fluctuation Measurements in Mixed Light Fields

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The statistical properties of light fields which are generated by superposition of laser light and chaotic light are investigated experimentally. The chaotic field is produced by scattering a laser beam at a moving ground-glass screen. The probability densities for the light intensity and their variances are measured and found to be in agreement with theory.

I. INTRODUCTION

IN his classical paper on the mathematical analysis of random noise, Rice[1] investigated the statistical properties of electrical signals in the presence of noise. Hodara[2] applied Rice's results to optical fields and calculated the probability densities of the field amplitudes in fields consisting of several laser modes and noise.

Lachs[3] and Morawitz[4] treated the statistical properties of mixtures of thermal and laser radiation quantum-mechanically on the basis of the coherent states of Glauber.[5]

On the experimental side the only papers on mixtures of thermal and laser light so far known are those by Armstrong and Smith.[6] They interpreted the intensity fluctuations of a GaAs laser near threshold by the interference of the laser radiation with the fluorescence light.

There are two main experimental difficulties in measuring the intensity fluctuations in the field of thermal light sources. The first one is the very short coherence time which in general is much smaller than the resolution time of the detectors. The second is the small intensity of available thermal sources; the fluctuations of the field intensity are covered by the shot noise of the detector.

These difficulties can be overcome by simulating the radiation field of a thermal source by the randomly scattered light produced by a laser beam passing a moving-glass screen.[7] The coherence time of such a quasi-thermal light source can easily be varied by the speed of the motion of the scatterer and can be made much larger than the resolution time of the detector. At the same time the intensity is so high that the shot effect can be neglected.

In this paper we present measurements of the intensity probability densities and their variances for radiation fields, which are generated by the superposition of laser light and the chaotic light of the quasithermal lamp.

II. EXPERIMENTAL ARRANGEMENT

Figure 1 shows the experimental arrangement. The linearly polarized beam of a single transverse-mode He-Ne laser is split into two beams at the plate M_1 of a Jamin interferometer. One of the beams is focussed on a rotating ground glass screen in order to produce a chaotic light field. This light field is superposed to the unchanged laser light by the second interferometer plate M_2. The intensity fluctuations of the superposition of the two fields are observed by means of the photomultiplier P. The beam intensities can be altered independently by two filters within the Jamin interferometer. The photocurrent of the photomultiplier P is observed by an oscilloscope. The standard deviation $\langle (I-\langle I\rangle)^2\rangle^{1/2}$ of the photocurrent is measured by a Siemens noise meter Rel 3U32d. In order to obtain the distribution function of the light intensity, the signal of the photomultiplier is chopped and analyzed by a 400-channel pulse analyzer.

III. THEORY

We describe the optical field[8] by the complex wave function V $(\mathbf{r},t)$ and use the current of the photomultiplier as a measure of the instantaneous light intensity $I=VV^*$. The intensity of the chaotic light in our experiments is very high. The mean number of photons per mode is always larger than 10^6, therefore the fluctuations of the photocurrent due to the shot effect are negligible compared to the mean current.

The chaotic field can be described by a Gaussian probability density[9]

$$p_{\mathrm{ch}}(X,Y)=(1/\pi\langle \mathcal{I}_{\mathrm{ch}}\rangle)\exp[-(X^2+Y^2)/\langle \mathcal{I}_{\mathrm{ch}}\rangle], \quad (1)$$

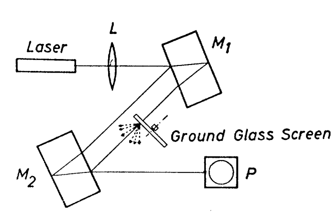

FIG. 1. Experimental arrangement. M_1 and M_2 are the beam splitting plates of a Jamin interferometer. The lens L focuses the laser onto the rotating ground-glass screen. P photomultiplier with a small pinhole.

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[2] H. Hodara, Proc. IEEE **53**, 696 (1965).
[3] G. Lachs, Phys. Rev. **138**, B1012 (1965).
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[6] J. A. Armstrong and A. W. Smith, Phys. Rev. Letters **14**, 68 (1965); A. W. Smith and J. A. Armstrong, Phys. Letters **16**, 38 (1965).
[7] W. Martienssen and E. Spiller, Am. J. Phys. **32**, 919 (1964); Naturwiss. **52**, 53 (1965).
[8] M. Born and E. Wolf, *Principles of Optics* (Pergamon Press, Inc., New York, 1959).
[9] L. Mandel and E. Wolf, Rev. Mod. Phys. **37**, 231 (1965).

where X and Y are the real and imaginary parts of the complex wave function $V(\mathbf{r},t)$ and $\langle \mathcal{I}_{\rm ch}\rangle$ denotes the mean intensity of the chaotic light. Introducing the intensity $\mathcal{I}=X^2+Y^2$ we get

$$p_{\rm ch}(\mathcal{I})=(1/\langle\mathcal{I}_{\rm ch}\rangle)\exp(-\mathcal{I}/\langle\mathcal{I}_{\rm ch}\rangle). \qquad (2)$$

For the probability density of the laser we assume[10]

$$p_l(X,Y)=\delta(X-X_l)\delta(Y-Y_l). \qquad (3)$$

The probability density for the superposed fields then is given by

$$p(X,Y)=(1/\pi\langle\mathcal{I}_{\rm ch}\rangle)\times\exp\{-[(X-X_l)^2+(Y-Y_l)^2]/\langle\mathcal{I}_{\rm ch}\rangle\}, \qquad (4)$$

a Gaussian distribution centered at the laser amplitude. Introducing polar coordinates ($X=Z\cos\theta$; $Y=Z\sin\theta$) and integrating over θ we get

$$p(Z)=(2Z/\langle\mathcal{I}_{\rm ch}\rangle)\times\exp[-(Z^2+Z_l^2)/\langle\mathcal{I}_{\rm ch}\rangle]I_0(2ZZ_l/\langle\mathcal{I}_{\rm ch}\rangle), \qquad (5)$$

where I_0 denotes the modified Bessel function of zero order. For the light intensity $\mathcal{I}=Z^2$ we have the probability density

$$p(\mathcal{I})=(1/\langle\mathcal{I}_{\rm ch}\rangle)\times\exp[-(\mathcal{I}+\mathcal{I}_l)/\langle\mathcal{I}_{\rm ch}\rangle]I_0(2(\mathcal{I}\mathcal{I}_l)^{1/2}/\langle\mathcal{I}_{\rm ch}\rangle). \qquad (6)$$

The nth moment of Z has been given by Rice[1]

$$\langle Z^n\rangle=\langle\mathcal{I}_{\rm ch}\rangle^{n/2}(n/2)!\,{}_1F_1(-n/2;1;-\mathcal{I}_l/\langle\mathcal{I}_{\rm ch}\rangle). \qquad (7)$$

The confluent hypergeometric function ${}_1F_1$ in Eq. (7) is a polynomial in $\mathcal{I}_l/\langle\mathcal{I}_{\rm ch}\rangle$. In particular, we calculate the variance of the light intensity

$$\langle(\mathcal{I}-\langle\mathcal{I}\rangle)^2\rangle=\langle Z^4\rangle-\langle Z^2\rangle^2=\langle\mathcal{I}_{\rm ch}\rangle^2+2\mathcal{I}_l\langle\mathcal{I}_{\rm ch}\rangle. \qquad (8)$$

Equation (8) is the classical analog of the formula (3.10b) of Lachs[3]; it does not contain the linear terms which are due to the shot effect.

IV. RESULTS AND DISCUSSION

Figure 2(a) shows the current of the photomultiplier P if it is illuminated by the chaotic light. The fluctuations with large probabilities for low intensities are typical for chaotic light (Eq. 2). The corresponding plot for laser light is drawn in Fig. 2(b); there are only small deviations from the mean intensity.[11] The superposition of the two light fields [Fig. 2(c)] yields much larger intensity fluctuations than those either in the laser or in the chaotic light field.

[10] In our experiment we measure interferences between the laser field and the chaotic field produced by the same laser. Therefore the phase fluctuations of the laser are eliminated and we can assume a phase-stabilized laser (Eq. 3). This assumption does not affect the generality of the results, because an additional randomness in the laser phase does not after the randomness of the phase differences.

[11] For a perfectly amplitude-stabilized laser one expects a sharp straight line in our approximation. The width of the line in Fig. 2(b) is due to the shot effect of the photoelectrons.

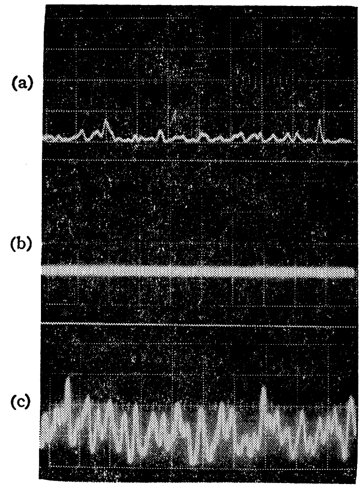

FIG. 2. Photocurrent of the detector P illuminated by (a) chaotic light, (b) laser light, (c) mixed light (chaotic light +laser light); time axis 10 msec/large unit.

The intensity distribution functions for such light fields are given in Fig. 3. On the left ordinate we have plotted the number of pulses per channel. The right ordinate indicates the normalized probability density times the mean intensity of the chaotic light. For pure chaotic light we expect an exponential distribution function. This function (Eq. 2) is drawn as a dashed line in Fig. 3(a) and is a rather good approximation to the experimental data.

Figures 3(b)–(d) show the distribution functions for mixed light fields. The ratios of the mean intensities of laser and chaotic light are 1, 3, 9, respectively. The distribution function of Fig. 3(d) corresponds to the intensity fluctuations of Fig. 2(b).

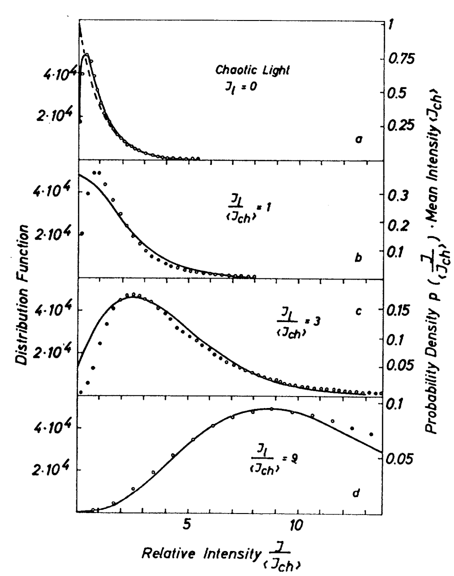

FIG. 3. Distribution functions (left ordinate) and probability densities (right ordinate) for mixed light fields with various ratios of laser intensity to mean chaotic intensity.

The full lines in Figs. 3(b)–(d) are calculated with Eq. (5). Again we find a good agreement with the experimental data. Deviations appear mostly at low intensities.

The standard deviation of the photocurrent for the chaotic light is shown in Fig. 4(a); the fluctuations are proportional to the mean intensity of the chaotic light. The relative intensity fluctuations $(1/\langle\mathcal{J}\rangle)\langle(\mathcal{J}-\langle\mathcal{J}\rangle)^2\rangle^{1/2}$ of chaotic light are 1.

In Fig. 3(b) we have plotted a measurement where the mean intensity of the chaotic light $\langle\mathcal{J}_{\mathrm{ch}}\rangle$ remains constant and the laser intensity I_1 is varied. For large laser intensities $(\mathcal{J}_1 \gg \langle\mathcal{J}_{\mathrm{ch}}\rangle)$ the fluctuations are proportional to the square root of the intensity of the laser light. The dashed line indicates a slope of $\frac{1}{2}$ for comparison. The full curve is calculated according to Eq. (8); the experimental data agree excellently with this curve.

The deviations for small light intensities between the calculated and measured probability densities in Figs. 3(a)–(c) occur for the following reason: In the theory of Sec. II point detectors with zero resolution time have been assumed. For real detectors and arbitrary radiation fields, however, we always measure $p(\mathcal{J})=0$ for $\mathcal{J}=0$. Because the light intensity never becomes negative the intensity averaged over the detector area and the resolution time is always larger than zero. Only a point detector could reveal the intensity zero.

For the case of pure chaotic light, the generalization of Eq. (2) for real detectors has been given by Mandel[12] and Troup[13];

$$\langle\mathcal{J}\rangle p(\mathcal{J}) = (N^N/(N-1)!)(\mathcal{J}/\langle\mathcal{J}\rangle)^{N-1}\exp(-N\mathcal{J}/\langle\mathcal{J}\rangle). \quad (9)$$

Here N is the number of coherence areas which are covered by the detector. For point detectors $(N=1)$ we get Eq. (2); the full curve in Fig. 3(a) is calculated with Eq. (9) putting $N=1{,}3$. This curve fits the measured data very well.

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[13] G. J. Troup, Proc. Phys. Soc. **86**, 39 (1965).

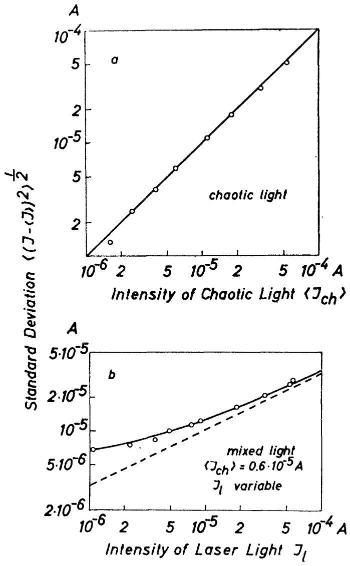

FIG. 4. Standard deviation of the photocurrent; (a) for chaotic light as a function of mean intensity, (b) for mixed light as a function of laser intensity. All intensities are measured as photocurrent in amperes.

Note added in manuscript. Since completion of this paper two contributions [F. T. Arecchi, Phys. Rev. Letters **15**, 912 (1965); F. T. Arecchi, A. Berné, and P. Bulamacchi, *ibid.* **16**, 32 (1966)] using a moving random scatterer for producing a Gaussian radiation field have appeared. These letters and our results supplement each other inasmuch as in our measurements the average photon occupation number was so high that shot noise could be neglected, while the other authors use photon numbers of order one.

Furthermore, C. Freed and H. A. Haus, Phys. Rev. Letters **15**, 943 (1965) and A. W. Smith and Y. A. Armstrong, Phys. Letters **19**, 650 (1966), report measurements using a laser below threshold as a Gaussian source. By the gain of the active medium the coherence time becomes so long, that observation times shorter than the coherence time are possible.

ACKNOWLEDGMENTS

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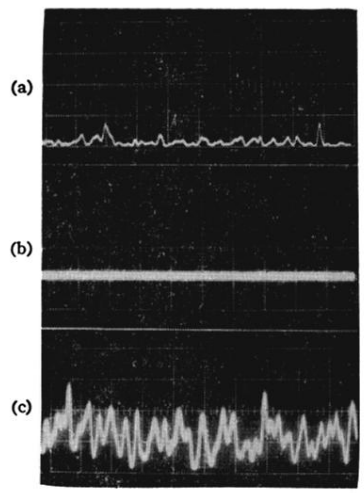

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