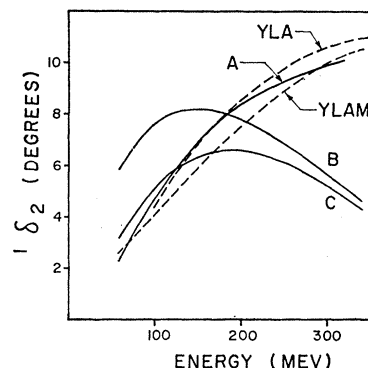


agreement with the 1D_2 phase shift obtained by the Yale group.

The 1D_2 phase shift for the nonlocal Yukawa potential is redrawn as curve A in Fig. 3, which also includes for comparative purposes, the primary and secondary results of the Yale group (YLAM and YLA). Curves B and C represent the results of Rojo and Simmons⁷ using a velocity-dependent potential without a repulsive hard core which fits the energy dependence of the S -wave phase shift.

Thus, it is apparent that a careful study of the data can yield much detailed information. The relative energy dependence of the 1S_0 and 1D_2 phase shifts shows that the interaction cannot be local. The shape of the potential likewise is seen to be more diffuse than the square well. Further, the speculation that the non-

FIG. 3. The singlet D -wave phase shift, $^1\delta_2$, plotted against energy. Curve A is the result of the nonlocal Yukawa calculation. YLAM and YLA are the results of the phase-shift analysis of the Yale group. Curve B and C are the results of Rojo and Simmons.



locality might so reduce the hard core as to render its role unimportant seems to be unwarranted.

The authors gratefully acknowledge the cooperation of the A.E.C. computing facility at New York.

⁷ O. Rojo and L. M. Simmons, Phys. Rev. **125**, 273 (1960).

Interference of K -Capture, Gamma Transitions through a Virtual State, and Inner Bremsstrahlung Transitions*

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(Received 11 March 1963)

The energy distribution of photons emitted during the transition from the ground state of Ni^{59} to that of Co^{59} has been calculated with allowance being made for interference between the K -capture, γ -ray transition through a virtual intermediate excited state of Co^{59} and the second forbidden K -capture transition, accompanied by inner bremsstrahlung, between ground states. The effect of the transition through the virtual state is to introduce terms in the photon energy distribution whose maxima are at higher energies than those resulting from inner bremsstrahlung transitions. In particular, the energy distribution for the leading interference term is similar to that used by Schmorak, who found that a better fit to his data was obtained by assuming that the virtual capture transition exists. Thus, the present calculation confirms Schmorak's method of analysis and, therefore, gives support for the existence of the virtual capture transition.

THE possibility of observing a combined beta (β)-decay, gamma (γ)-ray transition through a virtual intermediate nuclear state has recently been investigated by Rose, Perrin, and Foldy.¹ By considering the phase-space factors and energy denominators entering into the transition probability, these authors concluded that the K -capture transition in Ni^{59} offers the best opportunity for the experimental verification of such an effect. This combined K -capture, gamma transition proceeds from the ground state of Ni^{59} to the first excited state of Co^{59} by a Gamow-Teller allowed transition and then to the ground state of Co^{59} by emission of $M1$ or $E2$ radiation, as illustrated in Fig. 1. This gamma ray has a continuous spectrum since energy is not conserved in the intermediate state. The detection of a gamma ray is not sufficient, however, to establish

that the virtual transition occurred, since the second forbidden K -capture transition between the two ground states is accompanied by inner bremsstrahlung. In fact, the two transitions are coherent, and interference

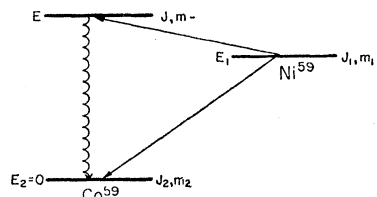


FIG. 1. Decay scheme of Ni^{59} . The transition through the virtual state goes by K capture from the ground state of Ni^{59} , labeled by the total angular momentum $J_1 = \frac{3}{2}$ with z component m_1 , to the first excited state of Co^{59} identified by the quantum numbers J_2, m_2 , and then by emission of $M1$ or $E2$ gamma radiation to the ground state of Co^{59} , labeled with J_2, m_2 . All states shown have negative parity. The values of the energies indicated by E_1, E , and E_2 are 1.076, 1.098, and 0 MeV, respectively.

* Supported by the U. S. Atomic Energy Commission.

¹ M. E. Rose, R. Perrin, and L. L. Foldy, Phys. Rev. **128**, 1776 (1962).

effects might considerably distort the spectrum from that expected for the two transitions separately.

On the basis of the results obtained by Rose, Perrin, and Foldy,¹ Schmorak² performed a careful measurement of the γ -ray spectrum from Ni⁵⁹. His analysis of these data indicated that the inner bremsstrahlung spectrum is slightly distorted near the end point. In prior work³ the quality of data was such as to allow only a confirmation of the second forbidden spectrum shape. Schmorak fitted his data with a function of the form

$$F(K, X) = I_{ib} + XI_{vc} \pm (2I_{ib}XI_{vc})^{1/2}, \quad (1)$$

where K is the ratio of the γ -ray energy k (with $\hbar=c=1$) to the maximum energy E_m available for the transition, X is a parameter representing the probability for the virtual capture transition, I_{ib} is the inner bremsstrahlung spectrum, and I_{vc} is the spectrum of virtual capture gamma rays. The positive sign is for constructive interference, whereas the negative sign indicates destructive interference. The form used for I_{ib} was

$$I_{ib} = K(1-K)^2[K^2 + \Lambda(1-K)^2], \quad (2)$$

where Λ is a ratio of nuclear matrix elements (in the notation of Uhlenbeck and Konopinski⁴) of the form³

$$\Lambda = [A_{ij} + 2\xi(\frac{1}{2}T_{ij} + iR_{ij})]^2 / [A_{ij} + \xi(\frac{1}{2}T_{ij} + iR_{ij})]^2, \quad (3)$$

with $\xi = Z\alpha/2R$ in units with $\hbar=c=1$ (R is the nuclear radius, α the fine structure constant, and Z the proton number). A more accurate form for I_{ib} including relativistic and Coulomb corrections⁵ was also used, but the results differed very little from those obtained by using (2), as these corrections are small above $K=0.3$, the energy region of the measurements. Finally, the form used for I_{vc} was

$$I_{vc} = CK^3(1-K)^2(\epsilon-K)^{-2}, \quad (4)$$

where C is a normalization constant chosen so that $\int I_{ib} dK = \int I_{vc} dK$, and ϵ is the ratio E/E_m , E being the energy of the first excited state of Co⁵⁹ relative to the ground state. The resulting best fit to the data obtained by Schmorak² was given by the parameter values $X=0.002$ and $\Lambda=0.49$ with destructive interference. For comparison, Saraf's³ best fit was given by $\Lambda=0.33$ (with X , of course, zero).

The purpose of the present paper is to present the results of a calculation of the interference between the two possible transitions. The amplitude for the transition through the virtual intermediate state is given by the perturbation theory formula

$$A_{vc} = \sum_m \langle J_2 m_2 | H_\gamma | J m \rangle \langle J m | H_\beta | J_1 m_1 \rangle (k-E)^{-1}, \quad (5)$$

where the states used in calculating the matrix elements

² M. Schmorak, Phys. Rev. **129**, 1668 (1963).

³ B. Saraf, Phys. Rev. **102**, 466 (1956).

⁴ E. J. Konopinski and G. E. Uhlenbeck, Phys. Rev. **60**, 308 (1941).

⁵ R. Marr, thesis, Harvard University, 1958 (unpublished). The approach used here is that of R. J. Glauber and P. C. Martin, Phys. Rev. **104**, 158 (1956); **109**, 1307 (1958). The author would like to thank Dr. Marr for sending him a copy of the thesis.

are labeled according to the notation of Fig. 1, k is the photon energy ($\hbar=c=1$), and $E=E_m+\Delta=\epsilon E_m$. The beta-decay interaction Hamiltonian is taken as the $V-\lambda A$ form⁶

$$H_\beta = (g/\sqrt{2}) \gamma_4 \gamma_\mu (1 + \lambda \gamma_5) \tau^{(-)} \psi_\nu^\dagger \gamma_4 \gamma_\mu (1 + \gamma_5) \psi_e, \quad (6)$$

where $\lambda = 1.19 \pm 0.04$, $g = 1.01 \times 10^{-5}/M^2$ (M =proton mass), $\psi_\nu^\dagger$ is the wave function for the created neutrino, ψ_e is the wave function for the destroyed K -shell electron, $\tau^{(-)}$ is an operator which changes a proton in the initial nuclear state function into a neutron, and the Dirac γ matrices are chosen in the convention of Rose.⁶ The repeated index summation convention is used. The interaction energy for the emission of the subsequent gamma ray is⁷

$$H_\gamma = -(e/M)(\mathbf{A} \cdot \mathbf{p} + \frac{1}{2}\mu \boldsymbol{\sigma} \cdot \mathbf{H}) = (e/M)(\pi k)^{1/2} \times [T_1^{-P}(\text{MD}) + PT_2^{-P}(\text{EQ})], \quad (7)$$

where e is the proton charge, $\mathbf{p}$ is the proton's momentum, μ is the magnetic moment of a nucleon in nuclear magnetons, $\boldsymbol{\sigma}$ is the nucleon spin operator, $\mathbf{A}$ is the vector potential of a (polarized) plane wave, and $\mathbf{H}$ is given by curl $\mathbf{A}$. The second form of Eq. (7) is obtained when $\mathbf{A}$ and $\mathbf{H}$ are expanded into multipole fields and only the lowest order terms consistent with the angular momentum and parity change are retained and expressed in terms of proper (under time reversal) irreducible tensor quantities. For the transition shown in Fig. 1, these lowest order irreducible tensors have rank one and two with projection $-P$ along the direction of photon emission, which is picked as the z axis. The values of P are $+1$ for right and -1 for left circular polarization. Explicitly,

$$T_2^{-P}(\text{EQ}) = i\mathbf{T}_{211}^{-P}(\mathbf{r}, \mathbf{p}) \quad (8)$$

and

$$T_1^{-P}(\text{MD}) = (\mu/\sqrt{2})\mathbf{T}_{110}^{-P}(\boldsymbol{\sigma}) - i\mathbf{T}_{111}^{-P}(\mathbf{r}, \mathbf{p}),$$

where, in the notation of Rose,⁸

$$\mathbf{T}_{L'L1}^{M'}(\mathbf{r}, \mathbf{B}) = \sum_{m'} C(1LL'; m', M'-m') \mathbf{Y}_L^{M'-m'}(\mathbf{r}) \mathbf{B}_{m'}. \quad (9)$$

The quantity $\mathbf{Y}_L^{M'-m'}(\mathbf{r})$ will be equal to r^L times the spherical harmonic of rank L with z component $M'-m'$ when it is multiplied by $[(2L+1)!/4\pi L!]^{1/2}$, and $\mathbf{B}_{m'}$ is the m' component of the vector $\mathbf{B}$ in a spherical basis.

The result of substituting (6) and (7) into (5) is

$$A_{vc} = \frac{eg\lambda(k\pi)^{1/2}}{M\sqrt{2}} \sum_m \langle J_2 m_2 | T_1^{-P}(\text{MD}) + PT_2^{-P}(\text{EQ}) | J m \rangle \times \langle J m | \boldsymbol{\sigma} | J_1 m_1 \rangle \cdot \frac{u_p^\dagger(\mathbf{q}) \alpha(1+\gamma_5) \phi_r(0)}{k-E_m-\Delta}, \quad (10)$$

⁶ R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193, (1958). For references to later work, review articles, and experiments one may refer to H. A. Weidenmüller, Rev. Mod. Phys. **33**, 574 (1961); or M. E. Rose, *Relativistic Electron Theory* (John Wiley & Sons, Inc., New York, 1961), Chap. 3, p. 105.

⁷ M. E. Rose, Proc. Phys. Soc. (London) **A67**, 239 (1954).

⁸ M. E. Rose, *Multipole Fields* (John Wiley & Sons, Inc., New York, 1955); and *Elementary Theory of Angular Momentum* (John Wiley & Sons, Inc., New York, 1957).

where the neutrino four-component row spinor for momentum $\mathbf{q}$ and spin ρ is $u_\rho^\dagger(\mathbf{q})$ and $\phi_r(0)$ is similarly a four-component column spinor for the K -shell electron evaluated at the nucleus.

The amplitude for the inner bremsstrahlung transition was calculated from a perturbation formula similar to (5); however, the Feynmann diagram technique is more straightforward, and the amplitude derived by Cutkosky⁹ with this method is essentially

$$A_{ib} = -ieg(\pi/k)^{1/2} u_\rho^\dagger(\mathbf{q}) \tilde{M}(1+\gamma_5) \{ [m_e + i\boldsymbol{\gamma} \cdot \mathbf{k} + \gamma_4(m_e - k)] / 2m_e k \} (\boldsymbol{\gamma} \cdot \boldsymbol{\epsilon}^*) \phi_r(0). \quad (11)$$

From right to left, the factors making up A_{ib} are a spinor for the initial K -shell electron (the contributions from other initial electron states are not of interest as the data were taken in coincidence with the K x ray), $\boldsymbol{\gamma} \cdot \boldsymbol{\epsilon}^*$ for the vertex at which the K -shell electron makes a transition to an intermediate state with the emission of a photon, a propagator (in curly braces) for the intermediate state electron, a beta-decay matrix element $\tilde{M}(1+\gamma_5)$ for the capture of the intermediate state electron, and a four-spinor for the final-state neutrino. In (11) m_e is the electron mass, $\mathbf{k}$ is the photon momentum, $\boldsymbol{\epsilon}$ is the polarization vector satisfying $\boldsymbol{\epsilon} \cdot \mathbf{k} = 0$, and $\tilde{M}$ is given by

$$\tilde{M} = \langle J_2 m_2 | [1 - \alpha(1 + \lambda\gamma_5) \cdot \alpha_L] e^{-i\mathbf{q} \cdot \mathbf{r}} [1 - i\mathbf{k} \cdot \mathbf{r} - \frac{1}{2}(\mathbf{k} \cdot \mathbf{r})^2 + i\xi\boldsymbol{\alpha} \cdot \mathbf{r} + \frac{1}{2}\xi\boldsymbol{\alpha} \cdot \mathbf{r} \mathbf{k} \cdot \mathbf{r}] | J_1 m_1 \rangle. \quad (12)$$

A subscript L , for lepton, is affixed to the Dirac matrix α to indicate that it comes from the lepton current part of the Hamiltonian. The last factor in square brackets is a matrix function which, when multiplied by an electron plane wave spinor, gives the wave function of the electron inside the nucleus without neglecting the spin-orbit coupling effect. This factor may be derived by evaluating the Sommerfeld-Maue wave function¹⁰ at R and matching it with a power series in r , $\sum_n a_n r^n$, for the wave function for $r < R$, and neglecting terms of order αZ relative to the leading terms. One further assumption is implicit in the form (11) for A_{ib} ; the binding energy E_b of the K -shell electron has been neglected compared to the photon energy. This is justified since the energy range of interest is $k > 0.3 E_m$, where $E_m = 1.076$ MeV $- E_b = 1.068$ MeV. Thus, the form of A_{ib} as given by Eq. (11) is essentially that expected in Born approximation without neglect of the "extraordinary" Coulomb effect.

The transition probability per unit time for the emission of a photon in the energy range between k and $k + dk$ with polarization P is given by

$$dW^P(k) = (2\pi)^{-5} \int \sum \delta(E_m - k - q) |A_{vc} + A_{ib}|^2 d\mathbf{k} d\mathbf{q}. \quad (13)$$

The integrations to be performed are over the direction of neutrino emission, the direction of photon emission,

⁹ R. E. Cutkosky, Phys. Rev. **95**, 1222 (1954). The author would like to thank Professor Cutkosky for a copy of his thesis.

¹⁰ H. A. Bethe and L. C. Maximon, Phys. Rev. **93**, 768 (1954).

and over the magnitude of the neutrino's momentum. The summation symbol stands for the sum over neutrino spin states, the sum over the azimuthal quantum number m_2 of the final nuclear state, for the average over spins of the initial state electron, and for the average over the azimuthal quantum number m_1 of the initial nuclear state.

The result of substituting (10) and (11) into (13) and performing the indicated integrations and summations may be summarized by three terms,

$$N^P(k) = dW^P(k)/dk = N_{vc}^P(k) + N_I^P(k) + N_{ib}^P(k), \quad (14)$$

where the terms on the right represent the contributions from the virtual capture branch, the interference terms between A_{vc} and A_{ib} , and the inner bremsstrahlung contribution, respectively.

The energy distribution of photons from the virtual capture process is

$$N_{vc}^P(k) = \left[\frac{eg\lambda |\phi(0)|^2}{\pi M} \right]^2 \frac{k^3(k - E_m)^2}{(k - E_m - \Delta)^2} \times R_\sigma^2 \left(\frac{1}{3} R_{MD}^2 + \frac{1}{5} R_{EQ}^2 \right). \quad (15)$$

Here, $\phi(0)$ is the value of the K -electron wave function (large component) at the nucleus and the R 's are reduced matrix elements in the convention of Rose,⁸

$$R_\sigma = (J \| \sigma \| J_1), \quad R_{MD} = (J_2 \| T_1(MD) \| J), \quad \text{and} \quad R_{EQ} = (J_2 \| T_2(EQ) \| J). \quad (16)$$

When R_σ^2 is expressed in terms of the $fT_{1/2}$ value for the virtual K -capture transition and R_{MD} and R_{EQ} are expressed in terms of the gamma-ray width of the excited state of Co⁵⁹, the result [Eq. (1)] of Rose, Perrin and Foldy¹ is obtained. As indicated by these authors,¹ the distribution goes as k^3 for small Δ and is independent of P .

The interference yields a contribution

$$N_I^P(k) = (1 + P) \frac{e^2 g^2 \lambda |\phi(0)|^2}{m M \pi^2} \frac{(k - E_m)^2}{k - E_m - \Delta} \times [a_1 k^3 + a_2 k^3 (E_m - k) + a_3 k^4], \quad (17)$$

where the a_i are defined by

$$\begin{aligned} a_1 &= R_\sigma [\xi \lambda \sqrt{3} (2\sqrt{2})^{-1} R_{\sigma rr}^{(2)} - (\xi/2) R_{rr} - R_{\sigma r}] \\ &\quad \times [(3\sqrt{2})^{-1} R_{MD} + 3(5\sqrt{10})^{-1} R_{EQ}], \\ a_2 &= R_\sigma [R_{rr} + (\lambda \sqrt{3}/\sqrt{2}) R_{\sigma rr}^{(2)}] \\ &\quad \times [(9\sqrt{2})^{-1} R_{MD} + (5\sqrt{10})^{-1} R_{EQ}], \end{aligned}$$

and

$$\begin{aligned} a_3 &= R_\sigma \{ R_{MD} [R_{rr}/(3\sqrt{2}) - \lambda R_{\sigma rr}^{(2)}/(2\sqrt{3})] / 5 \\ &\quad + R_{EQ} [R_{rr}/\sqrt{2} - \lambda R_{\sigma rr}^{(2)}/(2\sqrt{3}) \\ &\quad + \lambda \sqrt{2} R_{\sigma rr}^{(3)}/3] / (5\sqrt{5}) \}. \end{aligned} \quad (18)$$

The reduced matrix elements for the second-forbidden K -capture transition between the two ground states are abbreviated in a manner similar to Eq. (16),

$$R_{rr} = (J_2 \| \mathbf{Y}_2(\mathbf{r}) \| J_1), \quad R_{\sigma r} = (J_2 \| i \mathbf{T}_{21}(\mathbf{r}, \alpha) \| J_1),$$

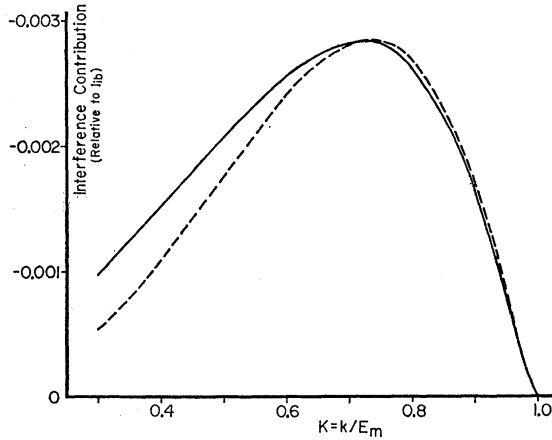


FIG. 2. Comparison of the leading interference term from Eq. (17) with that used by Schmorak. The dashed line is a plot of the leading interference term between the inner bremsstrahlung transition and the transition through the virtual state. This curve is normalized to the same maximum value as Schmorak's interference term $-(2I_{ib}XI_{ve})^{1/2}$, which is given by the solid line.

and

$$R_{\sigma rr}^{(L)} = (J_2 \| \mathbf{T}_{L21}(\mathbf{r}, \sigma) \| J_1), \quad (19)$$

where the irreducible tensors are defined by Eq. (9). The quantity $i\mathbf{T}_{211}(\mathbf{r}, \sigma)$ may be replaced by its non-relativistic limit¹¹ $(-i/M)\mathbf{T}_{211}(\mathbf{r}, \mathbf{p})$, obtained by applying a Foldy-Wouthuysen transformation. As a result of the definitions, all reduced matrix elements in (16) and (19) are real.

The calculated inner bremsstrahlung energy distribution is

$$N_{ib}^P(k) = (1+P)(eg|\phi(0)|/m\pi)^2 \times kq^2[b_1(b_1q^2/3 + b_2kq^2/9 - b_3q^3/15) + b_4(b_4k^2/3 - b_3k^2q/9 + b_2k^3/15) + d], \quad (20)$$

where the b_i are

$$b_1 = \xi R_{rr} + R_{\sigma r} - \xi\lambda(\sqrt{3}/\sqrt{2})R_{\sigma rr}^{(2)},$$

$$b_2 = (\lambda\sqrt{6})R_{\sigma rr}^{(2)} - 2R_{rr},$$

$$b_3 = (\lambda\sqrt{6})R_{\sigma rr}^{(2)} + 2R_{rr},$$

and

$$b_4 = (\xi/2)R_{rr} + R_{\sigma r} - \lambda\xi(\sqrt{3}/2\sqrt{2})R_{\sigma rr}^{(2)}. \quad (21)$$

The quantity d corresponds to a third order (in $1/\xi$) term,

$$d = \{ (k^4 + q^4 + 10k^2q^2/3)[R_{rr}^2 + (\lambda R_{\sigma rr}^{(2)})^2 + (\lambda R_{\sigma rr}^{(3)})^2] + 2(kq^3 + qk^3)[2R_{rr}^2/3 - (\lambda R_{\sigma rr}^{(2)})^2] \} / 30, \quad (22)$$

and is probably not reliable since corrections to the larger terms due to the simplifying assumptions in A_{ib} are expected to be of this same order of magnitude. The dominant terms in the inner bremsstrahlung spectrum are the $q^2 = (E_m - k)^2$ and k^2 terms in the square brackets of Eq. (21). These yield Eq. (2) when multiplied by kq^2 , the allowed spectrum factor, and when the identification $\Delta = b_1^2/b_4^2$ is made.

¹¹ M. E. Rose and R. K. Osborn, Phys. Rev. **93**, 1315 (1954).

In the spirit of the approximation leading to Eq. (2) for the inner bremsstrahlung spectrum, the corresponding leading term in the interference, Eq. (17), is the a_1 term with an energy dependence

$$k^3(k - E_m)^2/(E_m + \Delta - k) \approx k^3(E_m - k),$$

where Δ has been neglected in comparison with E_m . The function $-K^3(1-K)^2/(\epsilon-K)$ is shown in Fig. 2 along with the interference term, $-(2I_{ib}XI_{ve})^{1/2}$ of Eq. (1), for the best fit obtained by Schmorak. The former is normalized to have the same peak value as the observed interference term. The solid line, $-(2I_{ib}XI_{ve})^{1/2}$, is seen to agree very closely with the dashed line, $0.0298 K^3(1-K)^2/(K-\epsilon)$, with $\epsilon = 1.028$, except at the lowest energies. At these energies, the latter is smaller in magnitude than the former because of the $\Delta K(1-K)^4$ term in I_{ib} , Eq. (2). It is, therefore, quite unlikely that the results of Schmorak's analysis will be changed by the incorporation of the more accurate form, Eq. (17), for the interference.

When Δ can be neglected compared to E_m , the interference terms of Eq. (17) all approach zero at E_m as $(E_m - k)$. The inner bremsstrahlung terms of Eq. (20), on the other hand, all go to zero at E_m at least quadratically in $(E_m - k)$, so the existence of the transition through the virtual intermediate state could, in principle, considerably distort the photon spectrum. However, the existence of an energy-dependent correction [Eq. (20)] to I_{ib} which varies as $k^4(E_m - k)^2$ might contribute a significant deviation above $K = 0.5$ from that expected with I_{ib} alone. The possibility exists, however, that the coefficient of this term, b_4b_2 will be approximately zero.¹² The influence of the various correction terms to I_{ib} will be investigated further.

A final point concerns the polarization dependence of $N_{ve}^P(k)$, $N_I^P(k)$, and $N_{ib}^P(k)$ as given by Eqs. (15), (17), and (20), respectively. The inner bremsstrahlung and interference contributions are seen to be 100% right circularly polarized while the virtual capture transition yields unpolarized photons. The detection of left circularly polarized photons would, therefore, establish that the virtual transition occurred. This conclusion is true, of course, within the approximations of the present calculation. A more exact treatment of inner bremsstrahlung⁵ indicates that the polarization dependence is much more complicated than simply $(1+P)$; however, there is no significant decrease in polarization until the photon energy is well below the present range of interest.

The author is grateful to Professor L. L. Foldy for suggesting this calculation and for many helpful discussions throughout the course of the work. He would also like to thank the members of the theoretical staff at Case Institute for several stimulating discussions.

¹² M. A. Nagarajan (private communication). Preliminary calculations indicate that various nuclear models will give the result that R_{rr} and $R_{\sigma rr}^{(2)}$ are approximately equal, with R_{rr} being slightly larger. This would imply near cancellation of the two terms in b_2 , Eq. (21).