THE AUDION AS A CIRCUIT ELEMENT.

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SYNOPSIS.—The functional equations for the three-element audion are used to deduce the actions taking place in any circuit containing the audion. The equations for small variations in the general network containing audions are developed and simple equivalent circuits to replace the tube are shown. Next the effects of the capacities between elements are worked out and also some examples of amplifiers and oscillation generators.

THE three-element thermionic tube has come into such general use in radio telegraphy and telephony as well as in wire telephony and other fields that it seems worth while at the present time to publish some of those relations, holding between its characteristics and those of the connected electrical circuits, which are used in the design of amplifying, oscillating and detecting systems.

In this paper will be considered only those actions which may be sufficiently well described by supposing the variations in plate and grid currents and voltages to be small. In this discussion the methods adopted in a previous paper[1] will be shown in their application to vacuum tube circuits, in which application, as a matter of fact, their greatest practical use has been found.

The first figure shows a conventional circuit drawing of an audion in which the grid and plate potentials, with filament potential as zero, are denoted by v and V respectively, while the corresponding currents are denoted by i and I. A discussion of the operation of this device is unnecessary, since several papers have appeared describing the construction and operation in considerable detail. It has been found that, as long as there is an excess of electrons supplied by the filament so that temperature saturation is maintained, the plate current is represented with considerable accuracy by a function of a *single* variable, thus:

$$I = F(V + \mu v + c),$$

in which c is a constant which depends partly upon the contact differences of potential of the elements in the tube and μ is another constant which is characteristic of the structure. In many forms which the device may take the plate current is very approximately proportional to the square

[1] PHYS. REV., Aug., 1917.

of the variable above, but for the purpose of this discussion it is not necessary to so limit it.

The grid current depends also upon the two potentials, but its functional form is not known. For this discussion it is sufficient to recognize that it is such a function and write it:

$$i = f(V, v).$$

The problem to be considered in this paper is now the following. An audion is connected into an electrical network in any manner, that is, its three terminals are joined by any system of impedances; certain grid and plate voltages are impressed upon it with the result that certain currents, determined by the functional equations above, flow to plate and grid; and then it is required to describe the small variations about

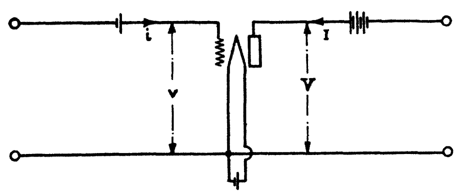

Fig. 1.

this steady state which will take place when the system is acted upon by arbitrary electromotive forces or otherwise disturbed. This is the problem of amplifiers and oscillation generators with the limitation that the amplitude of oscillation shall be small. While this limitation is serious in some cases, for example if it is desired to find the maximum power output of the device, still a great deal of information concerning the operation of the actual device may be obtained by thus studying the ideal case, which, in fact, is the only one which lends itself to simple and general treatment from the point of view of electrical networks.

In Fig. 2 is shown the audion connected into the network. In this figure the impedances Z_1, Z_2, Z_3, may be of any form whatever and are not restricted to single conducting paths as indicated. They are supposed only to be such circuits that it is possible to write an equation of the form $ZI = e$, connecting the current entering the terminals and the E.M.F. producing this current when the circuit is isolated. Certain electromotive forces, E, are supposed acting in each branch and the three circuits are coupled to one another by mutual impedances, M_{jk}, between impedance j and impedance k. These mutual impedances are defined, as usual, by the statement that a unit current in branch j induces an E.M.F. M_{jk} in branch k. Both self and mutual impedances are in the general case operators, functions of $p = d/dt$, as used in the paper above cited.

If in this network the three currents are chosen as shown, we may wirte down the network equations in terms of them as below:

$$Z_1(I_1 + I_3) + v = E_1 + M_{12}(I_2 - I_3) - M_{13}I_3$$
$$Z_2(I_2 - I_3) + V = E_2 + M_{12}(I_1 + I_3) + M_{23}I_3$$
$$Z_3I_3 + V - v = - M_{13}(I_1 + I_3) + M_{23}(I_2 - I_3).$$

These equations are of course exact and hold for all conditions; however, because they are not linear, their solution is somewhat complicated and it is preferable to obtain the first and approximate idea of the behavior of the network by considering only small changes in the currents

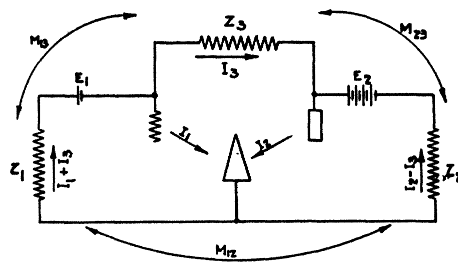

Fig. 2.

from their steady or mean values. This amounts to considering current variations over a part of the characteristic curve so small that it may be taken to coincide with the tangent at the given operating point and consequently the solutions obtained will not describe any effects which depend upon the curvature of the characteristic, such effects, for example, as the detection of signals.

Suppose then that in the departure from the steady values I_1 changes to $I_1 + x_1$, etc., these changes being small and produced by superposed electromotive forces e_1, etc. We shall then have

$$\delta v = \frac{\partial v}{\partial I_1}x_1 + \frac{\partial v}{\partial I_2}x_2, \qquad \text{etc.}$$

Substituting these values in the network equations, there will result a set of linear differential equations, with independent variable t, of the type:

$$Z_1(x_1 + x_3) + \frac{\partial v}{\partial I_1}x_1 + \frac{\partial v}{\partial I_2}x_2 = e_1 + M_{12}(x_2 - x_3) - M_{13}x_3$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

which set may be conveniently represented in the following tabular form:

x_1	x_2	x_3	
$Z_1 + \dfrac{\partial v}{\partial I_1}$	$- M_{12} + \dfrac{\partial v}{I_2 \partial}$	$Z_1 + M_{12} + M_{13}$	e_1
$- M_{12} + \dfrac{\partial V}{\partial I_1}$	$Z_2 + \dfrac{\partial V}{\partial I_2}$	$- Z_2 - M_{12} - M_{23}$	e_2
$- M_{13} + \dfrac{\partial V}{\partial I_1} - \dfrac{\partial v}{\partial I_1}$	$- M_{23} + \dfrac{\partial V}{\partial I_2} - \dfrac{\partial v}{\partial I_2}$	$Z_3 + M_{13} + M_{23}$	e_3

These are the network equations for small disturbances but it is convenient to throw them into another form which takes account of the physical meaning of some of the terms. To do this, note that $\partial v/\partial I_1$ is evidently the resistance, r, of the grid circuit under the given operating conditions, and $\partial V/\partial I_2$ is the resistance, R, of the plate circuit under the same conditions. From the characteristic equations we find by the usual methods, the further relations:

$$\frac{\partial v}{\partial I_1} = \frac{F'}{J} = r, \qquad \frac{\partial v}{\partial I_2} = \frac{-1}{J}\,\frac{\partial f}{\partial V},$$

$$\frac{\partial V}{\partial I_1} = -\frac{\mu F'}{J} = -\mu r, \qquad \frac{\partial V}{\partial I_2} = \frac{1}{J}\,\frac{\partial f}{\partial v} = R,$$

where

$$J = F'\left(\frac{\partial f}{\partial v} - \mu\frac{\partial f}{\partial V}\right).$$

Now since $\partial V/\partial I_1$ is the E.M.F. induced in the plate circuit through the tube by unit current in the grid circuit and rx_1 is the corresponding increment, δv, in grid potential, it follows from the above relations that one action of the tube is to introduce into the plate circuit an effective E.M.F. equal to μ times the grid voltage variation.

Also, since $\partial v/\partial I_2$ is the E.M.F. introduced into the grid circuit through the tube by unit current in the plate circuit, we find that another action of the tube is to introduce the voltage

$$\frac{\partial v}{\partial I_2}x_2 = -\frac{1}{J}\,\frac{\partial f}{\partial V}x_2 = -Rx_2\frac{\partial f/\partial v}{\partial f/\partial V} = -\delta V\cdot\frac{\partial f/\partial V}{\partial f/\partial v}$$

into the grid circuit. Thus the two mutual impedances of the tube are in general different in the two directions, which accounts for the amplifying properties of the device as explained in the paper already mentioned. This reverse action, from plate circuit to grid circuit, is often ignored in discussions of the audion and it is assumed that, since the plate current is a function of $V + \mu v$ only, the action of the device is summed up in the statement that the grid is μ times as effective as the plate in producing current changes. This action is, however, the only one if, for example, the grid current is independent of plate potential: there is then no reaction upon the grid circuit (neglecting capacity from grid to plate, which will be considered later) and the tube is a purely unilateral device. Since in practice this condition often holds very approximately, it is often legitimate to neglect $\partial v/\partial I_2$ and consequently to consider the tube as a device which, for small current changes, simply introduces into the

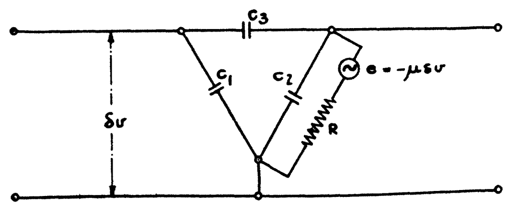

Fig. 3.

plate circuit (just where will develop later) an E.M.F. equal to μ times the change in grid potential. When this condition holds the last equations are still further simplified into:

x_1	x_2	x_3	
$Z_1 + r$	$-M_{12}$	$Z_1 + M_{12} + M_{13}$	e_1
$-M_{12} - \mu r$	$Z_2 + R$	$-Z_2 - M_{12} - M_{23}$	e_2
$+M_{13} - (\mu + 1)r$	$-M_{23} + R$	$Z_3 + M_{13} + M_{23}$	e_3

Equivalent Circuit of the Tube.—When the grid current does not vary apprecaibly with plate voltage it is possible to use a very simple equivalent circuit for the audion itself. This circuit is shown in Fig. 3 in which the resistance R of the plate circuit is shown between plate and filament and the three condensers, c_1, c_2, c_3 represent respectively the internal capacities between grid-filament, plate-filament and grid-plate. These capacities must obviously be taken into account at sufficiently high frequencies. Under the given conditions r is effectively infinite. Located in the impedance of the plate circuit is an alternator whose E.M.F. is equal to μ times the voltage, δv, produced across the grid-filament terminals. If this circuit is worked out it will be found that it is the exact equivalent of the actual tube under the given conditions. The feature of the circuit is the location of the fictitious E.M.F. between filament and plate, and the network will serve for the solution of tube problems as long as the electromotive forces impressed upon the network are small enough not to violate the conditions. The use of this equivalent avoids the necessity of setting up the complete equations.

It is obvious that, as a closer approximation, an equivalent circuit may be drawn in which another alternator is used to represent the reaction upon the grid circuit, but for practical use it has been found that this complication is in most cases unnecessary. Except at the very highest frequencies this network may be still further simplified by omitting the condenser c_2 since its capacity is but a few micro microfarads and it is shunted by the resistance R. The other condensers are, how-

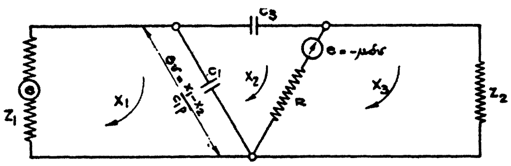

Fig. 4.

ever, very important, even at moderate frequencies, and this will be shown by two examples.

1. *The Input Impedance of an Audion Amplifier.*—Fig. 4 represents the equivalent circuit of an audion for frequencies below a few million and with the grid maintained negative with respect to the filament so that the input *resistance* is infinite. The output circuit is closed through an impedance Z_2. If there were no connection between the plate and grid circuits the impedance of the amplifier, as measured from the input terminals, would be simply that of the condenser c_1. The grid-plate capacity, however, modifies this result greatly. This impedance will be worked out and to do this the network equations, in terms of the mesh currents x_1, x_2, x_3 are given below with an E.M.F. e in the input circuit. p represents d/dt as usual and the fictitious E.M.F. in the tube resistance is μ times the voltage drop, $(x_1 - x_2)/c_1p$, over the condenser c_1. The sign is such that a positive grid tends to increase the plate current.

$$
\begin{array}{ccc|c}
x_1 & x_2 & x_3 & \\
\hline
Z_1 + \dfrac{1}{c_1p} & -\dfrac{1}{c_1p} & 0 & o \\[2ex]
-\dfrac{1+\mu}{c_1p} & \dfrac{1+\mu}{c_1p} + \dfrac{1}{c_3p} + R & -R & o \\[2ex]
\dfrac{\mu}{c_1p} & -R - \dfrac{\mu}{c_1p} & R + Z_2 & e
\end{array}
$$

If D is the operational determinant of this system and D_{11} the minor of the element $(1, 1)$ the input impedance is

$$Z = D/D_{11} \quad \text{with} \quad Z_1 = 0,$$

and this works out to be

$$Z = \frac{1}{c_1 p} \frac{1 + Wc_3 p}{1 + Wc_3 p + \frac{c_3}{c_1}\left(1 + \mu - \mu \frac{W}{Z_2}\right)},$$

in which W is equal to $RZ_2/(R + Z_2)$, the impedance of R and Z_2 in parallel. Note that if the grid-plate capacity is zero, this reduces to $1/c_1 p$ which is the impedance of the grid-filament condenser. This impedance is in the operational form; to find the impedance to steady alternating currents of frequency $n/2\pi$, p is to be replaced by in where i is the imaginary unit.

To save space only a few consequences will be noted, since it is easy to draw conclusions from the formula. In the first place, even at low frequencies (when the terms containing p as a factor are negligible) the impedance is charged into

$$\frac{1}{c_1 p} \frac{1}{1 + \frac{c^3}{c_1}\left(1 + \mu - \mu \frac{W}{Z_2}\right)}; \quad p \text{ small.}$$

In some cases this impedance only remotely resembles a capacity reactance; for instance if the output impedance, Z_2, is inductive, the impedance may have a negative resistance component. This means that the amplifier is likely to sing under these conditions, and in fact the tendency of a tube to oscillate "through its internal capacities" has often been observed.

At extremely high frequencies the formula shows that the impedance again approaches a capacity reactance, as it obviously should do since then the condenser c_1 practically short-circuits the current entering the input terminals. It is instructive to work out the impedance under various simplifying assumptions, and in the case of amplifiers which sing the solution of the difficulty will often be found in this manner.

2. *Amplification at High Frequencies.*—As a second example of the use of the equivalent circuit of the tube, suppose the circuit of Fig. 4 is to be used to produce in the impedance Z_2 an amplified current under the influence of the generator e in the input circuit. It is clear from inspection that for very high frequencies the condenser c_3 will have such a low impedance that the amplifying action of the tube will be seriously impaired. To get a quantitative measure of the effect of varying the frequency, consider what is meant by amplification. If the tube were a perfect unilateral amplifier a current x_1 in mesh 1 would produce a certain E.M.F., e_3, in mesh 3, while a current x_3 in mesh 3 would produce no E.M.F. in mesh 1. On the other hand, if the network were a simple

transforming one (no amplification), the same E.M.F.'s would be produced by the same currents in either direction through the network. Hence, defining the two mutual impedances in the usual way as

$$M_{13} = \frac{e_3}{x_1}, \qquad M_{31} = \frac{e_1}{x_3},$$

it is clear that for a perfect unilateral amplifier $M_{31}/M_{13} = 0$, while for a perfect transforming network this ratio is unity. The above ratio of the two mutual impedances is therefore a measure of the failure of the tube as an amplifier.

From the network determinant, already used, we find this ratio to be

$$\frac{M_{31}}{M_{13}} = \frac{D_{31}}{D_{13}} = \frac{1}{1 - \dfrac{\mu}{c_3 Rp}},$$

or for simple harmonic currents of frequency $n/2\pi$:

$$\frac{M_{31}}{M_{13}} = \frac{1}{1 + \dfrac{i\mu}{Rc_3 n}}.$$

It will be seen that for $c_3 = 0$ this ratio is zero, as it should be, and for c_3 infinite it is unity. Also, since the frequency appears in the ratio, the amplification will be different for different frequencies, so that the tube will distort and tend to eliminate higher frequencies.

With certain tubes and for certain frequencies for which the ratio $\mu/Rc_3 n$ is comparable with unity, the amplification is greatly impaired. This difficulty may, however, be avoided at any given frequency by the device of shunting the grid-plate capacity with an inductance of suitable value to make the impedance between grid and plate infinite at the given frequency. In doing this it is of course necessary to put in series with the inductance a large condenser to avoid conductive connection of grid and plate. It will be found that this method permits greater amplification at one frequency but it is open to the objection that variation of frequency is not convenient. Of course care must be taken not to couple this antiresonant mesh with others.

The Network as an Oscillation Generator.—In my Physical Review paper the general theory of oscillation generators was given and it was shown that the possibility of sustained free oscillation of a network depends upon the existence of certain types of unsymmetrical terms in in the characteristic determinant. The vanishing of the determinant then determines both the frequencies of all the possible modes of free oscillation and also the relations which must hold among the circuit constants in order that the oscillations shall be sustained (or have given damping). Since the solution for any oscillation generator requires only the ability to write down the network equations and find the roots of the determinant, it is hardly necessary to go into the detailed solution for any special case. As an example, however, the network of Fig. 5, purposely simplified to avoid long equations, will be handled by the method of the simplest equivalent circuit. The plate-grid capacity is

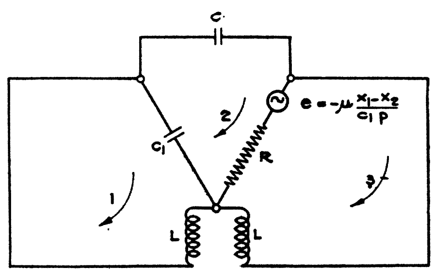

Fig. 5.

included in the capacity C and the mutual inductance of the two coils is m.

Choosing the mesh currents as shown, the network equations are

$$\begin{array}{ccc|c}
x_1 & x_2 & x_3 & \\
\hline
\frac{1}{c_1 p} + Lp & -\frac{1}{c_1 p} & -mp & 0 \\
-\frac{\mu + 1}{c_1 p} & \frac{\mu + 1}{c_1 p} + \frac{1}{Cp} + R & -R & 0 \\
mp + \frac{\mu}{c_1 p} & -R - \frac{\mu}{c_1 p} & Lp + R & 0
\end{array}$$

Note that these are the same as the last set with the addition of the transformer coupling. Now if we require that the oscillations be sustained, p must be taken equal to in, in which n is 2π times the frequency of oscillation. Then the vanishing of the determinant, which is complex, will require that the real and imaginary parts separately shall vanish, that is, that the even and odd powers of p shall separately be zero. As explained in the paper above cited, this determines both the frequencies of all the modes of oscillation and the circuit adjustments which must be made to permit the oscillations.

It will be noticed that in this determinant all the even powers of p appear in the coefficient of R; consequently by combining rows and columns until R appears in but one element, the even powers of p will be contained in the minor of that element, which minor, equated to zero, will determine the frequencies of sustained oscillation. This process leads to the frequency equation:

$$(L^2 - m^2)p^4 + \left(\frac{L}{C} + 2\,\frac{L-m}{c_1}\right)p^2 + \frac{1}{Cc_1} = 0,$$

and there are therefore two possible frequencies of sustained oscillation, corresponding to the two values of $p^2 = -n^2$. Whether or not these oscillations will actually take place depends upon whether the circuit constants are adjusted to make the odd powers of p vanish. This second equation, obtained by equating to zero the imaginary part of $D = 0$, is:

$$(L^2 - m^2)\left(\frac{c_1}{C} + \mu + 1\right)p^2 + \frac{L + \mu m}{C} = 0.$$

If, when one of the values of p^2 obtained from the first equation is substituted in this, it is satisfied, the oscillation corresponding to that value of p will take place. This fact explains a peculiarity of this kind of circuit, namely the common experience that in changing the frequency of an oscillator circuit the frequency may suddenly jump to some entirely different value for a small change in the constants. The reason is that in the process of changing the frequency the circuit constants are so changed that the relation holding among them becomes more nearly correct for another oscillation which would not before satisfy that relation. For example, in circuits of the type above, the most stable frequency is, for most ordinary adjustments, approximately that determined by the transformer and condenser C alone; but for some relations of L, m, μ, etc., the circuit will not maintain this frequency but will pass to the other. It is possible to design a network in which, as a constant is varied, the frequency will jump discontinuously from one value to others. The converse problem is to design an oscillation circuit in which the condition for sustaining a given oscillation holds for all variations of the element which is used to vary the frequency. Such a circuit will be stable over the whole frequency range.

Returning to the particular circuit above, it will be seen that if the capacity c_1 is very small the frequency of sustained oscillation will be that for which

$$n^2 = \frac{1}{2(L-m)C},$$

namely that corresponding to the single resonant circuit of the transformer and condenser C. The second equation then shows that this oscillation will be sustained if $\mu = 1$. If the inductances of the two windings had not been taken equal, μ would have turned out to be a function of L_1, L_2, m. It is worth noting that this circuit will oscillate even if there is no mutual inductance provided the circuit constants are properly adjusted. It then becomes identical with the amplifier, discussed above, with an inductive load.

It is probably unnecessary to mention the fact that the solution for all three-mesh oscillation generators may be derived directly from the determinant appropriate to this one by simply inserting the proper impedances. The solution for any case is theoretically just as simple, but practically the algebra is usually troublesome when there are more than three degrees of freedom. An exception is the case of so-called "iterative networks" which have periodic structures and certain other symmetrical networks.

As has been said, these methods of treating the audion in a circuit are approximate only, because of the limiting assumptions made, but considerable use of them in connection with experimental circuit work has shown that they are useful. The problem of finite amplitudes cannot be handled so generally, but when it is necessary to consider such refinements special methods are available which, it is hoped, may be given in another paper.

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