scattered into the region between two cones separated by a fixed angle increases as the angles of the cones increase.

When the data were thus modified, the number of electrons detected at each of the four angles was plotted against the angle, and the area under the curve was measured to give the integrated total for each of the five electron energies considered.

RESULTS AND CONCLUSIONS

Figures 2 and 3 show the relative agreement between experiment and the Brysk calculation for one-half and four inches of aluminum, respectively. The curves were normalized at 6 Mev. It is estimated that the uncertainty in the experimental points displayed in these figures is about 5% resulting from statistical inaccuracies in taking the data and from the latitude possible in drawing the curves of number *vs* angle under which the area had to be measured. Figure 4 illustrates the variation in the number of electrons of various energies per unit solid angle as a function of angle.

It is interesting to note that the agreement is quite good for both of the thicknesses of aluminum. The theory does not hold for the non-equilibrium region but the empirical weight function is apparently adequate to give agreement in this region.

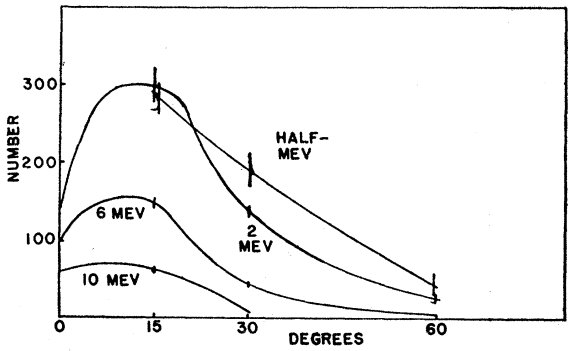

FIG. 4. The relative number of electrons plus positrons per unit solid angle plotted *versus* angle for one-half, two, six, and ten Mev electrons.

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Kinematic Viscosity of Helium II*

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By observing the time of formation of the meniscus in a cylindrical vessel suddenly given a rotation, it has been found possible to determine, directly, the kinematic viscosity of both He I and He II. At the angular velocities used (~15 rev/sec) it appears that, for He II, all the liquid rotates rather than the normal component alone.

WE have recently tried the following experiment with helium II. A cylindrical glass container closed at the bottom and with a 140-micron hole at the top (on the axis) is suddenly given a large angular velocity about the axis. This vessel is partially filled with He II. As is known,[1,2] the steady-state condition of the free surface is a parabola, as in a classical liquid.

In our case we measure the distance between the tip of the forming parabola and the original free surface (before the vessel is set in motion) as a function of time. Correcting for the small loss of liquid, due to creep and evaporation, we observe that a plot of $\log(\Delta Z)$ *versus* time yields, over many experiments, three distinct types of curves (Fig. 1). Here ΔZ is defined as $Z_\infty - Z(t)$, where $Z(t)$ is the distance from the original free surface to the tip of the parabola at any time t and Z_∞ is the steady-state distance. These three curves apparently represent the most usual "modes" of the system, and of these the single straight line (Curve C) occurs with considerably greater frequency than the others. The experimenter has no apparent control, in a given run, over the mode which is generated. However a change in speed, by as much as a factor of two, does not appreciably change the slope of the single straight line.

In this connection we also observed the phenomenon reported by Andronikashvili and Kaverkin[2] wherein the meniscus is not always truly parabolic. Sometimes a small "tip" is observed to form on the lowest portion of the parabola. When this tip forms as soon as the system is set in rotation, a curve such as B results;

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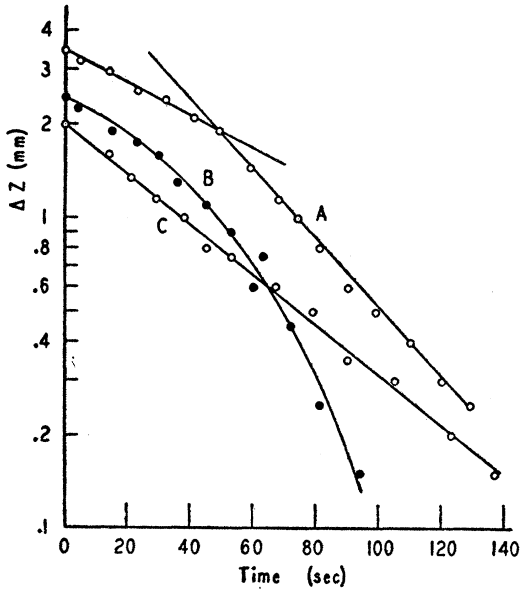

FIG. 1. Semilog plot of ΔZ *versus* time. In all three cases the radius of the cylinder was 3 mm and the angular speed 14.5 rev/sec. The temperature for curves A and B was 1.5°K; that for C was 1.8°K.

when, however, the tip suddenly appears after a lapse of time, we obtain a curve such as A. When no tip at all forms, we obtain the more usual "stable mode" curve C. In agreement with the above authors, we have also observed cases wherein the tip became progressively unstable and the paraboloid broke down into a vortex reaching to the bottom of the beaker. However, in disagreement with their observations we observed all these effects in both He I and He II.

The conditions under which the experiment is performed are such that the radial and vertical components of the fluid velocity are much smaller (by a factor of at least 100) than the angular velocity. This is a consequence of the small viscosity of liquid helium. It follows that the Navier-Stokes equation is greatly simplified, and, treating the liquid as "one fluid," we find:

$$\Delta Z = A \exp(-\alpha_1^2 \nu t/c^2), \tag{1}$$

where $\alpha_1 = 3.83$ is the first root of the first-order Bessel, ν is the kinematic viscosity, c is the radius of the cylinder, and A is a constant. If we now identify $c^2/\alpha_1^2\nu$ from Eq. (1) with our longest observed time constant τ (Curve C, Fig. 1), we can compute the kinematic viscosity of He II, a quantity which has not hitherto been measured directly. Actually Eq. (1) is the first term of a series, but it turns out that for times of the order of τ all terms except the first are negligible.

As a check on the method, we made some measurements in helium I. Here, unlike the situation with He II, all methods for measuring the viscosity coefficient agree

TABLE I. Summary of the experimental data.

T (°K)	$\langle\tau\rangle_{Av}$ (sec)	ν cm²/sec	μ' micro-poise	μ'' micro-poise	μ micro-poise
1.1	30.5	2×10^{-4}	0.6	29	20.6
1.3	30	2	1.5	30	15.4
1.5	35.4	1.7	3	25	13.6
1.8	42	1.4	7	21	12.7
2.1	42	1.4	16	21	17.4

and hence ν is reliably known. Our agreement is only in order of magnitude. Thus, at 2.25°K our measured value is 2.2×10^{-4} cm²/sec, as against 1.7×10^{-4} cm²/sec computed from the viscosity data of Bowers and Mendelssohn.[3] A method approaching the one we are using has been used previously for water, and here also only rough agreement has been found.[4] Nevertheless, as will emerge, for He II this accuracy appears good enough for our purposes.

The kinematic viscosity (ν) is defined as the ratio of viscosity coefficient (μ) to density (ρ). However, owing to the two-fluid nature of He II, there are two possibilities since we can use either the total density (ρ) or the normal-component density (ρ_n). Thus the normal-component viscosity may be computed from the observed ν either as

$$\mu' = \rho_n \nu = \rho\nu(\rho_n/\rho), \tag{2}$$

or as

$$\mu'' = \rho\nu. \tag{3}$$

In the vicinity of 1°K, μ' would be of the order of one percent of μ'' and they would be about equal near the λ point.

Table I is a summary of our results together with the latest values for μ (in the last column) due to Heikkila and Hollis-Hallett.[5] In computing μ' we have used the values of ρ_n/ρ as determined by Andronikashvili.[6] For each temperature, the value of $\langle\tau\rangle_{Av}$ in Table I is the arithmetic mean of five or six separate determinations and individual values differ occasionally from $\langle\tau\rangle_{Av}$ by as much as 25%. Nevertheless, it is clear that the correct density to use for the present transient case is the total density (ρ), as was found by Osborne[1] for the steady state situation. This suggests that, at the rotational speed we are using ($\omega \cong 15$ rev/sec), both the normal and superfluid components of the helium are in motion and implies some sort of viscous interaction between them.

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