an isotopic spin $T$ (=0, 1, or 2) state of the two pions. $\bar{\sigma}_T$ is the average of $\sigma_T$ over the momentum distribution of the pion cloud.

Since the wavelength of the incident pion in the center-of-mass system of the two pions is still large ($\sim 2\hbar/m_\pi c$) even at 900-Mev incident energy, only $S$ and $P$ wave interactions of the two pions are sufficiently strong to explain the large pion-nucleon cross sections in the 1-Bev region.

Assuming $\bar{\sigma}_0=\bar{\sigma}_2$, we obtained $\mathcal{P}\bar{\sigma}_1$ and $\mathcal{P}\bar{\sigma}_0$ from the experimental values of $\sigma(p,\pi^\pm)$. (See Fig. 1.) $\mathcal{P}\bar{\sigma}_1$ is much larger than $\mathcal{P}\bar{\sigma}_0$ and shows a pronounced peak at about 900 Mev.[3] Applying the one-level formula to the $p$-wave pion-pion scattering ($T=1$),

$$\sigma_1=12\pi\lambda\!\!\!^{-2}(\Gamma/2)^2/[(\epsilon-\epsilon_0)^2+(\Gamma/2)^2], \tag{3}$$

where $\lambda\!\!\!^{-}$ and $\epsilon$ are the wavelength and the energy of the incident pion in the center-of-mass system of the two pions, and neglecting the internal motion of the pion cloud, we can reproduce the $\mathcal{P}\bar{\sigma}_1$ curve between 500 Mev and 1100 Mev, if $\mathcal{P}\approx 40\%$, $\epsilon_0\approx 155$ Mev, and $\Gamma=50$–$90$ Mev.

The Doppler effect due to the internal motion of the pion cloud brings an additional width into the $\mathcal{P}\bar{\sigma}_1$ curve. For simplicity, we consider the case where the width due to this Doppler effect is larger than $\Gamma$, replacing Eq. (3) by

$$\sigma_1=12\pi\lambda\!\!\!^{-}{}_0^2(\pi\Gamma/2)\delta(\epsilon-\epsilon_0), \tag{3'}$$

where $\lambda\!\!\!^{-}{}_0$ is the wavelength corresponding to the resonance energy $\epsilon_0$. $\mathcal{P}\bar{\sigma}_1$ is given by averaging Eq. (3′) over the momentum distribution of the pion cloud. If we take a momentum distribution of the form[4] $q^2\exp(-\alpha q^2/m_\pi^2)$, the experimental $\mathcal{P}\bar{\sigma}_1$ can be obtained by taking $\alpha=6$–$10$ and $\Gamma=55$–$40$ Mev with $\epsilon_0\approx 155$ Mev and $\mathcal{P}\approx 40\%$. Because of the large effect of the internal motion of the pion cloud, only a very narrow distribution of the pion cloud can be consistent with the experimental $\mathcal{P}\bar{\sigma}_1$.[5]

Our model predicts also the following features of pion-nucleon scattering. Since the probability of the pion cloud having more than one meson is small, the inelastic pion-nucleon scattering is predominantly one-pion production until the incident pion energy becomes high enough to produce a pion by a pion-pion collision. If the latter becomes appreciable when the pion energy in the center-of-mass system of the two pions is larger than 340 Mev, then the two-pion production in the pion-nucleon scattering becomes appreciable for a larger energy than 1.7 Bev.

If the pion-nucleon scattering occurs only through the $T=1$ state of the two pions, the ratios of the various types of inelastic scattering are

$$\sigma(p+\pi^-\to p+\pi^-+\pi^0):\sigma(p+\pi^-\to n+\pi^-+\pi^+):\sigma(p+\pi^-\to n+2\pi^0)=1:1:0$$

and

$$\sigma(n+\pi^-\to n+\pi^-+\pi^0):\sigma(n+\pi^-\to p+2\pi^-)=1:0$$

when there is no final interaction between the pions and the nucleon, while

$$\sigma(p+\pi^-\to p+\pi^-+\pi^0):\sigma(p+\pi^-\to n+\pi^-+\pi^+):\sigma(p+\pi^-\to n+2\pi^0)=17:26:2$$

and

$$\sigma(n+\pi^-\to n+\pi^-+\pi^0):\sigma(n+\pi^-\to p+2\pi^-)=13:2$$

when either one of the two pions composes a $T=\frac{3}{2}$ isobar state with the nucleon after the two-pion interaction. These ratios are consistent with the experimental data.[6] The ratio between the elastic scattering and the inelastic scattering depends on the strength of the interaction between the pions and the nucleon after the two-pion interaction.

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[1] Cool, Madansky, and Piccioni, Phys. Rev. **93**, 637 (1954); Shapiro, Leavitt, and Chen, Phys. Rev. **92**, 1073 (1953).

[2] F. J. Dyson [Phys. Rev. **99**, 1037 (1955)] has proposed independently a similar model which assumed the presence of a resonance state of the two pions in $T=0$ state. We are obliged to him for sending us his manuscript before publication and for comments he made on this paper. Also we are very grateful to Dr. M. Ross who informed us about Dyson's work.

[3] The arbitrariness of the assumption $\bar{\sigma}_0=\bar{\sigma}_2$ does not change these qualitative conclusions about $\mathcal{P}\bar{\sigma}_1$ as long as only $S$ and $P$ wave interactions are considered.

[4] This is proportional to the probability of a pion in the cloud for having a momentum between $q$ and $q+dq$. The momentum distribution can be taken to be spherically symmetric, since the direction of spin of the nucleon is averaged out.

[5] This difficulty was pointed out to the author by Dr. O. Piccioni, who had suggested a similar model several years ago, and by Dr. M. Ross. If the strong pion-pion interaction works only when the bound pion has a very small momentum, this difficulty might be solved. This possibility was suggested to the author by Professor F. J. Dyson.

[6] W. D. Walker (private communication).

## Spatial Extension of the Proton Magnetic Moment from the Hyperfine Structure of Hydrogen

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Until recently the measurement of the hyperfine splitting of the ground state of hydrogen, used in conjunction with the theoretical formula,[1] has provided the most accurate means of ascertaining the value of the fine structure constant, $\alpha=e^2/\hbar c$. In this note, we propose to employ the formula, together with an independent measurement[2] of $\alpha$, to estimate a "magnetic radius" for the proton.

The doublet separation, $\Delta\nu_H$, can be represented to within a few parts per million by the equation

$$\Delta\nu_H=\frac{16}{3}\alpha^2 cR_\infty\frac{\mu_p}{\mu_0}\left\{1+\frac{m}{M}\right\}^{-3}\left\{1+\frac{\alpha}{2\pi}-2.97\frac{\alpha^2}{\pi^2}\right\}$$

$$\times\{1+\tfrac{3}{2}\alpha^2\}\left\{1-\alpha^2\left(\frac{5}{2}-\ln 2\right)\right\}\left\{1-\frac{\alpha}{\pi}\frac{m}{M\mu_p}\right.$$

$$\times\{[3-\tfrac{3}{4}(\mu_p-1)^2]\ln\frac{M}{m}-\tfrac{1}{8}(\mu_p-1)^2\}$$

$$\left.-\frac{9}{4}\frac{\alpha}{\pi}\frac{m}{M\mu_p}(\mu_p-1)^2\ln\frac{2k_0}{M}\right\}\{1-\delta\}, \quad (1)$$

where $\mu_p$ is the proton magnetic moment and $\mu_0$ the the electron Bohr magneton.

The leading factor is the Fermi formula,[3] whereas the factors in brackets represent successively the reduced mass correction, half the gyromagnetic ratio of the electron, the Breit correction, the term which takes account of the distributed character of electrodynamic effects, the contribution of proton recoil, and finally, a hitherto unknown fractional correction, $-\delta$, arising from the structure of the proton. The calculation of the penultimate factor[1] presupposed that the proton is characterized as a point charge with an anomalous point magnetic moment. The result depends, although not sensitively, on structure in that a cutoff, $k_0$, is required for the terms in which the proton interacts exclusively via the anomalous part of its moment. Equation (1) is a double power series in $\alpha$ and $m/M$, in which the corrections of relative order $\alpha$, $\alpha^2$, and $m/M$ can be certified.[4] On the other hand, previous treatments have lacked the criteria for determining the accuracy of the term of relative order $\alpha m/M$ and for distinguishing recoil and structure effects.

By employing an approach in which the electron-proton system is studied by means of the relativistic two-body equation and all the interactions of the proton, other than with the electromagnetic field, are treated phenomenologically, we have been able to justify the use of Eq. (1). In particular we have been able to demonstrate that the proton recoil factor of Eq. (1) contains the only term of relative order $\alpha(m/M)\ln(M/m)$, although there may be additional terms of order $\alpha(m/M)$ too small to worry us here. Moreover, we have established that the main contribution to $\delta$ is obtained by considering the proton as a distributed source of magnetic dipole field with magnetization $\mu_p f(r)$, where $f(r)$ is a spherically symmetric function with total strength unity,

$$\int d\mathbf{r}\, f(r)=1, \quad (2)$$

to insure the correct value of the magnetic moment. It follows that $\delta$ has the value

$$\delta=2\bar{r}/a_0, \quad (3)$$

where $a_0=\alpha(\hbar/mc)$ is the Bohr radius and

$$\bar{r}=\int d\mathbf{r}\, rf(r) \quad (4)$$

is the first moment of the distribution of magnetization.

In general, we may expect $\bar{r}$ to lie between the nucleon and meson Compton wavelengths. Thus the relative correction to Eq. (1) may be as large as $5\times10^{-5}$. On the other hand the direct measurement of fine structure has yielded a value of $\alpha^{-1}=137.0365\pm0.0012$, accurate to $1\times10^{-5}$. If we remark that $\alpha(m/M)\ln(M/m)$ is of relative order $3\times10^{-5}$, we realize that Eq. (1) is sufficiently precise to permit an estimate of $\bar{r}$. Inserting the recent experimental value of $\Delta\nu_H$,[5] the value of $\alpha^2 cR_\infty$ from the fine structure measurement,[2] and all other atomic constants from the review article of DuMond and Cohen,[6] we obtain

$$(3.26\times10^{-6})\ln(2k_0/M)-2\alpha\bar{r}/a_0=(2.6\pm20.0)\times10^{-6}, \quad (5)$$

where the uncertainty follows from the error in $\alpha^2$. For any reasonable value of $k_0$, say $k_0\sim M$, we may assert that the value of $\bar{r}$ is bounded from above by the inequality

$$\bar{r}\leq 2.5(\hbar/Mc), \quad (6)$$

as determined by the experimental error in Eq. (5). The upper bound thus established lies barely within the limits of error for the proton "size" deduced from the electron-scattering result of Hofstadter and McAllister.[7] It should be recalled, however, that the latter experiment measures $\langle r^2\rangle_{\mathrm{Av}}$ for a combination of the charge and magnetization distributions.

A detailed account of the calculations is in preparation and will be submitted later for publication.

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2 Dayhoff, Triebwasser, and Lamb, Phys. Rev. **89**, 106 (1953).

3 References to all significant previous work will be found in reference 1.

4 Corrections of relative order $\alpha^3\ln\alpha$ of purely electrodynamic origin have been omitted. These may be expected to be of the same order as $\alpha(m/M)$.

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7 R. Hofstadter and R. McAllister, Phys. Rev. **98**, 217 (1955).