THE STABILITY OF RESIDUAL MAGNETISM.

BY N. H. WILLIAMS.

A. THEORETICAL.

I. Analogies between Elastic and Magnetic Phenomena.

THE magnetization of iron under the influence of a magnetizing field is often considered as a phenomenon analogous to the distortion of an elastic body under stress. The elastic strain theory has been built upon this analogy. The intensity of a magnetizing field at a point is defined as the force per unit pole at the point. It is a vector quantity and in a homogeneous medium is a continuous function, hence it is permissible to think of the flux of the vector per unit area perpendicular to the vector as a measure of the field intensity; thus it becomes analogous to stress in a body which is in a condition of strain. The substance in which the magnetic field is established undergoes a change which is also a vector point function and its flux per unit area is called the magnetic induction at a point, and is analogous to the strain in a body under stress. The ratio of the magnetic induction to the intensity of the magnetizing field is called the permeability of the medium and is analogous to the reciprocal of an elastic modulus.

The deformation of an elastic body occurs in approximate accordance with Hooke's law; but beside this approximate proportionality of stress and strain, other effects are observed such as elastic after-effect or hysteresis, plastic yield, permanent set due to repeated application of the load, and the increase of elastic limit with over-strain.

A magnetic phenomenon which is analogous to elastic after-effect may be observed when a demagnetizing field is applied to magnetized iron or steel. After the first partial demagnetization due to the field, there is a further slight demagnetization which is too slow to produce an appreciable effect upon a ballistic galvanometer. There is a similar time lag when the field is in such a direction as to increase the induction, and the effect is much more apparent in soft iron than in steel. These facts have been mentioned by Lord Rayleigh[1] and by Ewing.[2]

[1] Phil. Mag., March, 1887.
[2] Phil. Trans., 1885, p. 569.

Plastic yield and permanent set are analogous to the magnetic phenomena resulting from any change of magnetic induction exceeding a limit beyond which the specimen fails to return to its former state upon being removed from the field.

It was first shown by Fromme[1] and afterward by Ewing[2] that repeated applications of a magnetizing field produce a higher residual induction than a single application. This is analogous to the fact that repeated application of the load upon an elastic body may produce a permanent set provided the stress approaches the elastic limit.

II. Definition of Terms.

It will be convenient to use the term elastic limit as applying to magnetic phenomena and meaning the maximum intensity of field which may be applied to a specimen without a permanent change of its induction. A *partial* demagnetization will be designated merely as a demagnetization and the word recovery is here used to represent the change of flux which occurs when the demagnetizing field is removed.

III. Purpose of the Work.

The purpose of this paper is to present some experimental work which suggests further analogies between elastic and magnetic phenomena and especially to determine for different kinds of iron and steel the relation between the recovery and the maximum residual flux.

IV. The Scope of the Work.

The experiments show an approximate proportionality between magnetic stress and strain and that the limit beyond which the iron fails to regain its former condition after partial demagnetization may under favorable circumstances reach a variation of magnetic induction as high as 2,000 lines per sq. cm.

It is shown that for hard steel the hysteresis, for changes within the elastic limit, is quite small and the analogy between magnetic hysteresis and mechanical hysteresis is made evident by comparing this work with work recently done upon the elastic properties of steel tubes.

The effect of repeated application of a *demagnetizing* field is found to be similar to that found by Fromme and Ewing for a *magnetizing* field, *i. e.*, many repetitions of a magnetic stress, which is equal to the elastic limit, produce a change of induction that is equivalent to a permanent set.

An analogy is found for the increase of elastic limit with over-strain.

[1] Pogg. Ann., 1875; Wied. Ann., IV., 1878.

[2] Phil. Trans., 1885, p. 570.

B. Experimental.

I. *Madelung's Experiment.*

Madelung[1] has published an account of an excellent piece of work on the magnetizing effects of high frequency currents. His method involved the use of the Braun tube in plotting variation of magnetic flux in steel wire with variation of field intensity caused by the discharge current from a condenser. Fig. 1 is a copy of one of his curves showing the demagnetizing effect of an aperiodic discharge. He plots what he calls the maximum hysteresis curve, *i. e.*, the curve resulting from carrying the magnetizing field to a very high value and then reducing it. It will be observed that the curve due to the discharge lies outside the maximum

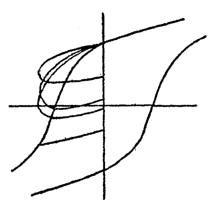


Fig. 1.

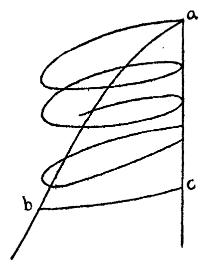


Fig. 1*a*.

curve. This is due to the effect of eddy currents in the steel, which set up a counter magnetizing field and cause a shift in phase of the actual magnetizing field. The curve obtained from the instrument gives the relation of the magnetic induction to the magnetizing field as it would be if not modified by eddy currents. The three curves of Fig. 1 were taken with wires of different size, the curve taken with small wire being the lowest one. Since the eddy current effects decrease with diminished size of wire, this lowest curve would be expected to conform more nearly to the maximum hysteresis curve than the others, as it does.

Fig. 1*a* shows the effect of repeated application of the demagnetizing impulse. The curve falls to lower values of the induction as the impulse is repeated and finally reaches a stationary position where it becomes a closed curve. However, it never falls to a position where its extreme point lies on the hysteresis curve, but the point remains outside the area of the curve. Madelung points out that this is an error due to the eddy currents and suggests that if the current were of long duration and its time rate of variation at all times small, the curve traced would be the maximum hysteresis curve down to the point *b*, and from there it would

[1] Annalen der Physik, 17, 1905, p. 861.

be the line bC. This is undoubtedly true for the first application of the demagnetizing field but the question arises: what would be the result of many repetitions of the demagnetizing impulse? A part of the experimental work herein described deals with this question. An effort has been made to reduce the errors due to eddy currents and end effects to a negligible magnitude and furthermore to deal with the whole matter in a quantitative way so that we may know to just what extent demagnetization is followed by recovery and how much hysteresis occurs.

II. General Method.

The method adopted in the experimental work was an oscillographic method devised by the author and so far as he is aware it has not been used elsewhere. The specimen to be studied was of steel wire in the form of a ring wound with primary and secondary coils. The secondary of a transformer operating on a sixty cycle current was connected in series with a storage battery and adjustment made so that the maximum value of the alternating E.M.F. was equal to the storage battery E.M.F. This gave a unidirectional pulsating current fluctuating between the limits of zero and a maximum value. This current was made to pass through the primary coil on the magnetized core in such a direction as to produce a demagnetizing field. A portion of this current operated one loop or vibrator of the oscillograph while the other loop was connected to the secondary consisting of a few turns and also wound on the steel core. The inductance in the secondary was negligible and hence the ordinate of the curve obtained by the oscillograph from this coil is proportional to the rate of flux variation while the ordinates of the current curve represent the instantaneous value of the magnetizing field. This latter statement is only true provided the results of eddy currents and end effects are negligible.

Proper calibration is necessary in order to obtain actual values of field intensity and rate of flux variation from these curves.

In order to obtain the curve whose ordinates represent the change of flux due to the demagnetizing field, the curve whose ordinates represent the time rate of change of flux is integrated graphically, the process being as follows: The curves as photographed by the oscillograph are transferred by means of a pantagraph to cross-section paper, the copy being drawn to double scale. The paper used was divided into twentieths of an inch. At the instant when the ordinate of the curve representing rate of flux variation is zero and increasing in the positive direction, the flux is a maximum. Directly above this point a, Fig. 2, at the top of the sheet a line bc is drawn across the quarter inch in which this point lies parallel to the axis of abscissæ. Across the next quarter inch a line *cd* is drawn from the extremity of the preceding line and its slope is made equal to the negative of the ordinate *mn* of the curve representing time rate of flux variation at the middle point of this quarter inch. Where this line ends, another begins and crosses the next quarter inch with a negative slope equal to the ordinate of the flux rate curve at the middle of this space. The process is continued until the curve for the cycle is completed. If the work is carefully done the ordinate of the curve, measured downward, represents the flux variation with considerable accuracy. If the ordinary hysteresis curve is wanted an alternating

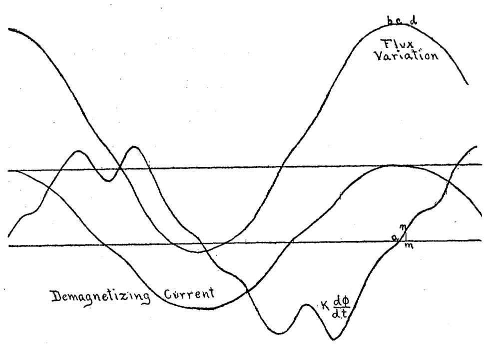
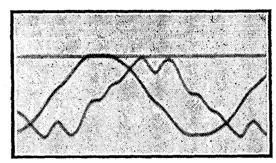


Fig. 2.



Oscillogram. The smooth curve represents the unidirectional primary current; the irregular curve the secondary current.

Fig. 2*a*.

current is used instead of the unidirectional pulsating current. The values of H, as computed from the ordinates of the current curve are plotted as abscissae and the values of the flux or the induction B, computed from the flux curve, are plotted as ordinates. Thus we have the hysteresis curve when the alternating current is used and the demagnetization and recovery curve when the pulsating current is used. A hysteresis curve obtained by this method is shown on Fig. 7, the specimen being piano wire in its normal condition.

The method has several advantages over the ballistic method for work involving small changes of B and H. It is possible to determine as many points on the curve as desired even when the curve represents very small changes. The apparatus is quite sensitive to small changes of flux while for large changes it is only necessary to reduce the number of secondary turns and put resistance into the circuit. For example, one curve was plotted in which the flux variation was only 114 lines per sq. cm. and it showed all the characteristics of the curves representing much larger changes; on the other hand hysteresis curves representing changes of magnetic induction of more than 30,000 lines per sq. cm. are easily obtained. The method also insures conditions giving a closed curve for the cycle since the impulse is repeated a large number of times before the photograph is taken and the cycle is traversed more than 1,000 times during the exposure of the sensitive paper.

The remaining details of the method were different for different specimens and will be mentioned in connection with the experiments to which they apply.

III. Calibration.

A ballistic galvanometer was used for a part of the work. It was calibrated by means of a current inductor in the usual way.

One mm. deflection was found to correspond to a flux variation of 32.6 lines of induction.

The same current inductor was also used in calibrating the oscillograph. A wire was used for the secondary coil of such a size that its resistance per turn was the same as that of the secondaries of the steel cores. This secondary was connected to the same loop of the oscillograph that was afterward used for the secondaries of the steel cores. The maximum flux through the coils was computed from the same formula as that used in calibrating the galvanometer. Calibration curves were taken with 25, 50, 75 and 125 turns respectively in the secondary. The unidirectional current was used here as in the work upon the steel cores. The flux curve in each case was obtained by graphic integration as previously described. The maximum ordinate of this curve corresponds to the flux as computed from the maximum current. The value in terms of lines of induction of one division of the cross-section paper is thus determined, *e. g.*, with 25 turns one division corresponds to 22.9 lines of induction. Since the maximum flux variation is used in this determination, it is evident that it must be computed from the maximum current rather than from the effective value.

Since a direct current instrument having its field furnished by a permanent magnet indicates the mean current flowing through it and since the mean value of an alternating current furnished by a transformer is zero, a direct current ammeter reads one half the maximum current when the current is unidirectional and fluctuates between zero and a maximum value. The instrument used was a Weston ammeter reading directly to .02 of an ampere and having large divisions so that one could estimate to .005 ampere with accuracy.

As an illustration of the accuracy of the process, two calibration curves for a secondary of 75 turns were taken at different times, one about two weeks later than the other and under conditions giving very different wave forms. The flux value of one division of the paper was 8.104 in one case and 8.089 in the other, showing a difference of two tenths of one per cent.

A separate calibration was made for plotting the hysteresis curves of the specimens. This became desirable since a secondary of only one turn was used and some resistance was inserted to reduce the induced current in the secondary circuit to a value that could be used with the oscillograph.

The magnetic flux per division of the cross-section paper as given by the calibration curves was as follows, the numbers in the first column being the number of secondary turns.

125 turns,	6.65 lines.
75	8.09
50	11.94
25	22.90

IV. Experiments with Specimen A.

The first specimen to be studied was of piano wire 1.03 mm. in diameter. It was in the form of a ring of mean circumference 25 cm. and cross-sectional area 1.4 sq. cm. A gap was cut in the ring about one cm. long and a plug of soft iron wire was made to fit it. The ring was heated to a red heat in an electric furnace and then plunged into cold water. The core was next insulated with tape and a secondary coil consisting of five sections of twenty-five turns each was wound upon it. The primary of 350 turns was wound outside the secondary.

The gap was made in the ring in order that it might be possible to determine at any time the residual flux in the core. This determination was made by means of the galvanometer and test coil previously calibrated. With the coil encircling the ring on the side opposite the gap, the plug was removed and the throw of the galvanometer noted. The coil was then quickly slipped off the ring and a second deflection observed. Since the calibration showed 32.62 lines of induction per mm. deflection, the total flux was found by multiplying 32.62 by the sum of the two deflections.

A series of photographs was taken for the glass-hard steel with maximum demagnetizing field ranging from 1.65 gauss to 34.5 gauss. The variation of induction in the first case was 114 lines of induction per sq. cm. while in the latter it was 1,960. In this latter case the remanence was 140 lines per sq. cm. and the application of the current caused a reversal of the magnetism, producing a negative induction of 1,820 lines per sq. cm. As the current was reduced to zero the magnetic flux returned to its former positive value. With somewhat smaller demagnetizing currents the magnetism was not reversed and the recovery on reducing the field to zero was from a value of the induction which was still positive to a higher positive value. For example, when the current was 1.56 amperes, giving a field of 27.4 gauss, the induction fell from 1630 to 60 and returned to 1,630 when the current became zero. The magnitude of these recoveries is appreciated when they are compared with the maximum residual magnetism observed in the specimen. This latter value was 8,036 lines per sq. cm.; the recovery of 1,960 is therefore 24.5 per cent. of the maximum residual induction.

The curves for this specimen were taken in order beginning with small demagnetizing current. The first application of the current produced a certain amount of permanent demagnetization. After many applications of the current the magnetism reached a stable condition in which the recovery was just equal to the demagnetization. Then the photograph was taken. Next a larger current was applied which produced a further permanent demagnetization and a new condition of stability. Then another photograph was taken. The method of determining the residual flux after each photograph was taken has been described. The removal of the plug before each observation introduced a demagnetizing field due to the ends and caused a marked decrease of the residual magnetism when the flux was at its high values. This effect decreases as lower values of the remanence are reached and vanishes when the flux becomes zero. These curves were plotted with magnetic flux as ordinates rather than the flux density or induction while the figures quoted above are values of flux density. The area of the specimen was 1.4 sq. cm. hence to obtain variation of induction from the curves the ordinates should be divided by 1.4.

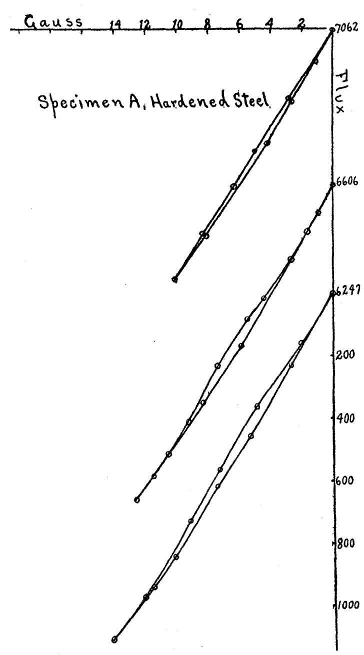


Fig. 3.

Figs. 2 and 3 are two of the series obtained from this specimen. The curves were all transferred to a single sheet (see Fig. 4) and properly placed with reference to the axes according to the residual flux determined after each was taken.

Data for Demag. and Recovery Curves of Specimen A.

Current.	No. of Secondary Turns.	Def. on Removing Plug.	Def. Due to Remaining Flux.	Remanent Flux.	Maximum *H*.	Gauss per Division.
.094	125	24.0	207	7,535	1.6537	.0438
.48	75	21.0	198	7,144	8.4446	.1206
.576	50	20.5	196	7,062	10.1336	.1842
.72	50	19.5	183	6,606	12.667	.2112
.80	50	18.5	173	6,247	14.074	.2346
.92	50	17.5	159	5,757	16.185	.234
1.00	25	16.0	146	5,284	17.593	.3824
1.18	25	14.0	122	4,436	20.76	.3844
1.32	25	12.0	99	3,621	23.222	.3776
1.40	25	10.0	88	3,197	24.63	.4561
1.56	25	7.0	63	2,283	27.445	.524
1.68	25	6.0	49	1,794	29.556	.5185
1.80	25	3.5	32	1,158	31.667	.5278
1.96	25	0	6	197	34.482	.6000

Data for Demagnetization and Recovery Curves Derived from the Current Curve and the Flux Variation Curve.

H = field intensity. ϕ = magnetic flux.

H	ϕ	H	ϕ	H	ϕ	H	ϕ
.24	22.3	.75	70	1.105	107	1.02	95
.48	49.9	2.08	168	2.763	233	2.95	238
.85	81.8	3.98	308	4.973	399	4.50	368
1.40	136.0	5.18	401	6.263	513	5.60	440
1.65	159.0	6.51	522	8.289	668	7.50	584
1.36	133.0	8.44	656	10.133	812	11.55	938
.70	68.0	6.93	559	8.289	668	12.67	1015
.48	46.2	5.43	437	6.080	501	10.60	862
		3.74	308	4.140	370	7.50	582
		2.65	219	2.763	239	5.58	438
		1.03	93			4.50	365
						1.89	155
2.111	167	1.872	191	1.912	160	1.53	130
4.93	370	4.446	406	4.589	389	3.84	321
7.27	573	7.956	668	8.413	664	7.88	641
9.15	740	10.070	859	10.707	870	11.92	916
11.96	979	13.104	1134	14.149	1164	14.61	1214
14.07	1110	16.180	1432	16.443	1374	18.45	1540
12.43	1014	13.104	1183	17.59	1466	20.76	1740
10.09	847	9.600	919	16.44	1374	18.07	1557
7.51	621	6.790	692	14.53	1213	15.00	1305
5.40	465	4.446	441	11.472	962	11.15	985
2.81	239	14.04	167	7.839	733	7.88	733
				5.162	504	4.23	378
				1.912	183	.87	114

H	ϕ	*H*	ϕ	*H*	ϕ
3.77	321	1.82	137	1.57	160
8.31	664	4.11	343	4.19	400
12.46	1054	8.66	710	9.43	836
15.86	1374	12.77	1053	14.67	1150
20.01	1740	16.41	1397	18.34	1534
23.26	1992	20.75	1786	23.31	1946
20.39	1786	24.86	2038	27.45	2198
16.61	1511	21.89	1866	23.80	1969
12.08	1145	18.24	1580	19.38	1649
8.68	847	13.68	1191	18.34	1237
4.91	458	9.12	893	10.18	962
		5.93	470	5.67	481
		1.82	149	1.05	137
1.80	183	2.64	251	4.5	378
4.67	492	7.38	618	9.9	801
10.37	939	13.20	1100	16.8	1340
16.07	1340	18.50	1511	22.8	1763
20.48	1800	23.75	2015	27.3	2255
25.93	2233	29.46	2427	34.5	2748
29.55	2496	31.67	2611	27.3	2358
25.93	2233	30.08	2473	22.2	1981
21.26	1855	26.64	2267	15.3	1488
15.54	1397	21.00	1855	9.9	1008
10.89	1031	14.98	1397	4.5	423
5.70	527	10.55	973		
1.39	137	4.22	424		

It will be observed that the extreme points of the demagnetization curves lie on a smooth curve. The determination of the relation of this curve to the maximum hysteresis curve was next undertaken. This portion of the hysteresis curve was determined ballistically and was found to lie outside the locus of the extreme points and as would be expected deviated widely from it near the upper end. This deviation was due to the end effects established when the plug was removed as previously explained. A second ballistic curve was traced under the same conditions that existed when the work on the oscillographic curves began, *i. e.*, the core was highly magnetized, then the plug was removed and replaced, thus the remanence was brought down to the same value at which it stood when the oscillographic work began. This second ballistic curve follows quite closely the contour of the locus of the extreme point of the demagnetization curve as is seen on Fig. 4. However, it lies outside of this locus as does the maximum hysteresis curve.

That the relation of these two curves is not due to error is shown by further work by the ballistic method. The specimen was first highly

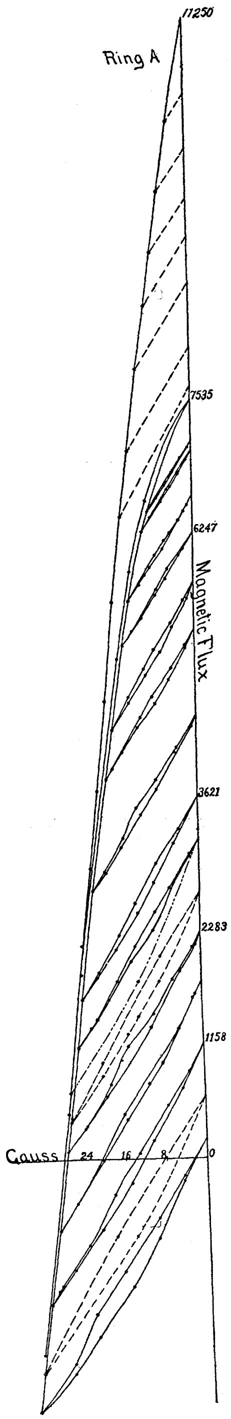


Fig. 4.

magnetized, then a certain demagnetizing current applied sufficient to bring the residual magnetism down to the region in which the oscillographic curves were obtained. Conditions would then be represented by some point on the maximum hysteresis curve. From this point the curve of recovery was traced ballistically, as many points being determined as seemed expedient. This curve was found to differ from that obtained by the oscillograph for the same demagnetizing current both in position and slope. The conditions differed in the two cases only in this: the ballistic curve was obtained the first time the steel was put through the cycle while the oscillographic curve was taken after several thousand repetitions of the cycle. It was next found that after opening and closing the circuit through the primary, a second ballistic curve approached more nearly the oscillographic curve. When the key in the primary circuit was opened and closed about a hundred times and the ballistic curve then traced, it was found that its extreme point fell almost exactly upon the locus of the extreme points of the oscillographic curve, and furthermore the slope and shape of the new curve agreed with those of the oscillographic curve. A number of cycles of demagnetization and recovery were traced ballistically. Some of them are shown on Fig. 4. in their relation to the other curves.

In plotting the hysteresis curve by the ballistic method, an interesting fact appeared regarding end effects. The residual induction after the application of a strong magnetizing field and before the plug was taken out was about 8,000 lines of induction per sq. cm. When the plug was removed the end effects caused it to fall to a value of 4,940 lines per sq. cm. The core was then remagnetized with the plug in place and a demagnetizing current applied. This current might be of any strength provided it did not reduce the induction below 4,940. The circuit carrying this current was then opened and a certain recovery ensued. Then the plug was removed and it was found that the residual induction was again 4,940. This constancy of the residual magnetism on open magnetic circuit persisted until a demagnetizing filed of 12.8 gauss had been applied and then suddenly the residual magnetism, with the magnetic circuit open, fell to lower values. The point at which it breaks down is the point where the recovery on removing this demagnetizing field of 12.8 gauss is equal to the demagnetization due to the ends when the plug is removed. Fig. 5 shows these relations in graphic form. Curve *A* represents the maximum hysteresis curve. The inercept between *A* and *B* is the recovery upon breaking the circuit. The intercept between *B* and *C* is the demagnetization following the removal of the plug. At the right of point *P* the demagnetization due to the ends exceeds the recovery while at the left of *P*, the recovery is

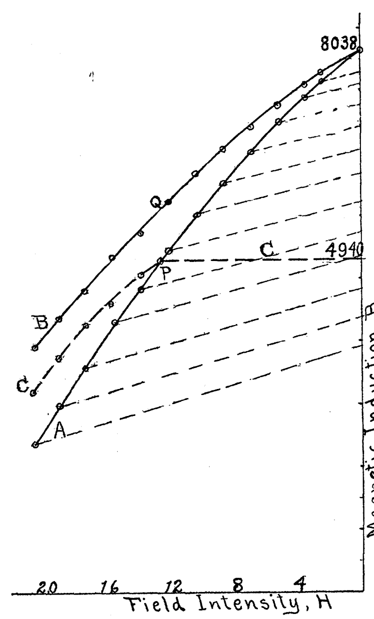


Fig. 5.

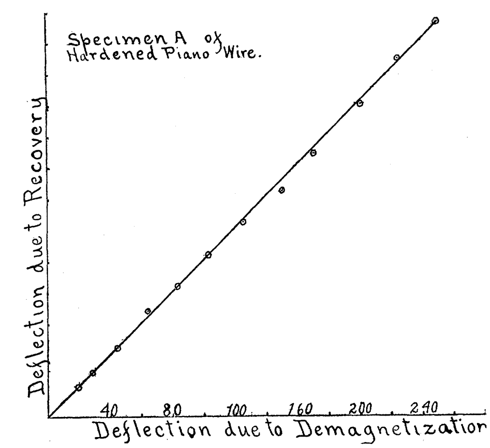


Fig. 6.

greater. Intercept *PQ* represents both demagnetization due to the ends and recovery after removal of the field and it is at this point that the residual magnetism with the gap open ceases to be constant. The interpretation of this horizontal portion of curve *C* seems to be this: that there is a welldefined upper limit beyond which the remanence of a steel core with exposed ends cannot be made to pass.

As has been mentioned, the recovery curves for a given specimen are practically parallel. For very hard steel the portion of the hysteresis curve with which we are concerned does not differ much from a straight line. If it were exactly a straight line, it would follow that the recovery would be strictly proportional to the difference between the maximum residual flux and the residual flux corresponding to the recovery curve. The line of Fig. 6 is presented to show how nearly this statement of proportionality approximates the truth.

In designing magnetic circuits, so much attention is usually given to reducing the magnetic reluctance of the joints that it seems desirable to find out if the reluctance of the joints had any influence upon the preceding experiments. A demagnetization and recovery curve was obtained by the oscillograph with the plug out of the circuit; thus the magnetic circuit contained an air gap about one centimeter long. The only marked difference between this curve and the others was a decrease in slope amounting to about thirteen per cent. This would lead to the conclusion that when the gap was filled with soft iron, the effect of the joints, upon the form or slope of the demagnetization and recovery curves would be insignificant.

V. Experiments with Specimen B.

The second specimen studied was a ring of no. 4 piano wire in its normal condition, *i. e.*, without heat treatment. It consisted of 1,215 wires each having a cross-sectional area of .0012 sq. cm., the aggregate area being 1.458 sq. cm. The mean length of the ring was 32.67 cm. The secondary coil consisted of five sections of 25 turns each. Outside the secondary, the primary coil of 350 turns was wound. The intensity of magnetic field within the metal was $\frac{4\pi \cdot 350\ i}{10 \cdot 32.67}$ gauss, i being the current in amperes. When a test coil of fourteen turns was used, one mm. deflection of the ballistic galvanometer indicated a change of flux of 32.62 lines or a change of flux density (B) of $32.62/1.458 = 22.37$. In dealing with softer specimens, which are much more susceptible to end effects, it seemed best to try to do the work without making any air gap in the magnetic circuit, hence this ring and two others of soft iron which were used were not cut.

Plate 7 shows the hysteresis curve by the oscillographic method when the maximum field intensity is about 62 gauss. The residual induction is seen to be 13,665. As a check upon this curve a determination of the residual inductance was made by the ballistic method using the same maximum magnetizing field and it was found to be 13,670. Such close agreement is probably the result of a fortunate observation as it is beyond the limits of accuracy that could reasonably be expected.

A series of demagnetization and recovery curves was obtained for this specimen by the oscillographic method. They differ in slope from those of the former series but otherwise show very little difference.

Since there was no opening in the magnetic circuit, the method used in the former case of determining the position of the curves with reference to each other and to the residual induction could not be employed. In the case of ring *A* the curves were placed each with its highest point at the point representing the residual flux actually measured after the curve was photographed and the curve through their extreme lower points was drawn. The results of that work would warrant us in taking the reverse process this time. The locus of the extreme points lies within the maximum hysteresis curve. A hysteresis curve that approaches the maximum hysteresis curve very closely can be obtained by the ballistic method but just where the locus of the extreme point lies with reference to this curve is a matter to be determined. This was done by means of a Koepsel permeameter. A field of 134 gauss was applied to a specimen of this wire, this value of field intensity being chosen in order to reproduce the conditions under which the ballistic curve was taken. The field was then reduced to zero leaving a remanence of 15,400 lines per sq. cm. A small demagnetizing field was next applied and the value of the flux density, *B*, observed from the instrument. Then the circuit through the magnetizing coil was opened and closed 50 times and another reading of *B* taken. The key was then opened and closed 50 times more and the value of *B* again observed. Next a larger demagnetizing field was applied and the process repeated. Fourteen points on this portion of the hysteresis curve were thus found and also fourteen on a curve lying within the hysteresis curve. This latter curve, it was assumed would represent the locus of the extreme points of the recovery curves provided the hysteresis curve were accurate. In using the permeameter it is necessary to correct the observeed curve by adding to its abscissæ the abscissæ of a shearing curve. No shearing curve

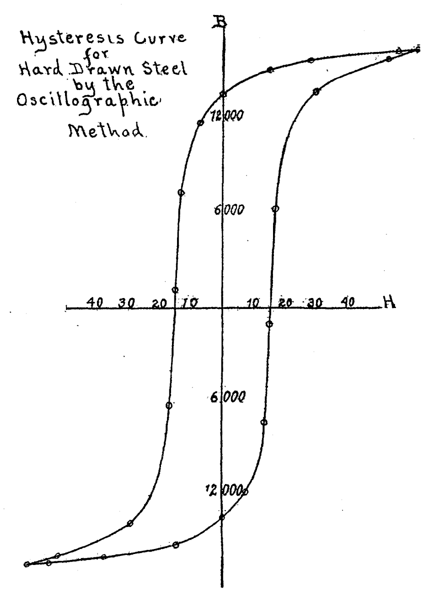


Fig. 7.

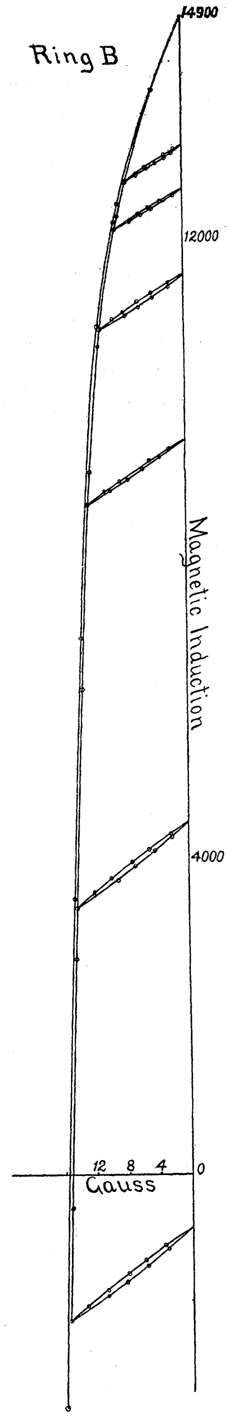


Fig. 8.

was at hand that could be assumed to be accurate for this specimen and so the ballistic curve shown in Fig. 8 was used. The curve of the extreme points of the recovery curves would lie within this hysteresis curve and it was assumed that the difference in ordinates of these two curves would be the same as the difference between the ordinates of the corresponding curves given by the permeameter. In this way the locus of the extreme points was determined. Both these curves are shown in Fig. 8. The oscillographic curves of demagnetization and recovery were now reduced to the same scale and placed upon the sheet in such a way that their extreme points fell upon the curve which lies within the hysteresis curve. The maximum recovery observed was 1,200 lines per sq. cm. and occurred in the region of zero residual flux while the maximum remanence was 14,900, thus the recovery is about 8 per cent. of the maximum remanence. The maximum recovery for the hardest steel was 24.5 per cent. of the maximum remanence. This large difference is due both to a smaller recovery and to a larger remanence in the second case.

Attention has been drawn to a number of analogies between elastic and magnetic phenomena. Bridgman[1] has recently done a piece of work on "The Collapse of Thick Cylinders under High Hydrostatic Pressure" that has furnished curves bearing a striking resemblance to the curves of Figs. 4 and 8. Fig. 13 is a copy of one of Bridgman's figures inverted for the purpose of comparison with the magnetic curves. The specimen is a hollow steel cylinder and the strain is measured by the change of internal volume. Three points of resemblance are to be observed: (1) The hysteresis, (2) the increase of elastic limit with overstrain, (3) the effect of repeated application of the same stress as shown near the bottom of the figure. The actual change of elastic limit with over-strain is of course much less than the apparent change, a large part of that appearing in the figure being due to change of dimensions of the specimen.

[1] Phys. Rev., January, 1912, p. 1.

The demagnetization and recovery discussed in the previous pages have taken place under conditions such that the metal was carried during each cycle to the limit of its elasticity for the particular remanence to which it returned. For lower values of remanence the elastic limit is higher. The question would naturally arise as to the effect of a demagnetizing field that did not approach the elastic limit. The shape of the curves already taken would lead one to expect to find the change of flux due to a given demagnetizing field very nearly independent of the residual flux. This would follow from the fact that magnetic stress and strain have been found approximately proportional. Lord Rayleigh[1] has shown that the change of induction resulting from a very small change of field intensity is independent of the magnetic state of the iron, or to quote his conclusion, "That the value of the susceptibility to small changes of force is approximately independent of the initial conditions as regards force and magnetization until the region of saturation is approached." He was dealing with field strengths ranging from .04 gauss to .000018 gauss. The oscillographic method was applied to this problem, using a much larger demagnetizing field. The results may be made clear by referring to two curves produced by the same field intensity. In one case the residual flux density was 12650 and in the other 500 lines of induction per sq. cm. The maximum field intensity was 7.27 gauss. The reduction in flux density from the higher remanence was 440 while it was 475 from the lower. This represents a difference of about 8 per cent., and means that the susceptibility to small changes of field intensity increases as the remanence decreases. Similar results were obtained by the ballistic method.

Data for Demagnetization and Recovery Curves of Specimen B.

$$\text{Max. } H = \frac{4\pi \cdot 350 \cdot i}{10 \cdot 32.67} = 13.46\, i.$$

i being the current.

Current.	No. Turns in Secondary.	Maximum H.	Gauss per Division.
.27	1.25	3.6342	.0512
.54	.50	7.2684	.1191
.54	.50	7.2684	.1483
.64	.75	8.616	.1325
.79	.50	10.6334	.1715
.92	.50	12.385	.1567
1.06	.50	14.2676	.1719
1.17	.25	15.75	.2045
1.13	.50	15.21	.1925

[1] Phil. Mag., 23, 1887, p. 225.

Data for Demagnetization and Recovery Curves of Specimen B.

Value of one division in gausses taken from preceding table. Value of one division in lines of magnetic induction determined by the calibration. H = field intensity. B = variation of induction.

H	B	H	B	H	B
.427	25	.476	23	.89	41
1.254	77	1.071	82	2.14	139
2.330	160	2.50	156	3.40	213
3.020	212	3.69	225	4.59	286
3.630	265	4.78	303	5.77	368
2.690	219	5.97	368	7.27	475
1.970	164	7.27	440	5.77	384
.947	77	5.35	327	4.37	303
.358	32	3.94	254	3.18	238
		2.91	180	2.07	156
		1.67	98	1.03	74
		.70	35		
1.06	58	1.95	110	1.88	122
2.78	150	4.13	240	4.39	262
4.19	228	5.90	344	6.11	393
5.51	322	7.70	483	8.15	540
7.50	439	9.06	573	10.18	667
8.62	511	10.63	704	12.38	835
6.62	408	7.35	532	9.40	647
5.24	319	5.64	426	7.13	516
3.84	233	3.93	311	5.33	377
2.32	139	1.79	172	3.13	221
.99	60			1.25	82
2.15	156	1.02	71	1.16	49
4.99	328	3.58	235	1.93	98
7.23	491	6.14	416	5.77	266
9.92	695	8.28	585	8.08	385
12.04	880	11.04	806	9.72	491
14.27	1064	13.50	1015	12.80	638
12.04	901	15.75	1194	15.21	761
8.94	737	10.84	871	12.03	610
6.88	573	8.59	699	8.18	454
4.30	360	5.83	487	6.45	361
2.06	180	3.07	267	1.35	74

VI. *Experiments with Specimen C.*

Specimen C is a ring of transformer iron of mean length 37.93 cm. and cross-sectional area 2.07 sq. cm. The primary coil has 325 turns, hence the magnetizing field is

$$\frac{4\pi \cdot 325\, i}{10 \cdot 37.93} = 10.767\, i.$$

One division deflection of the galvanometer corresponds to a change of induction of 15.76 lines per sq. cm. when there are fourteen turns in the test coil.

When the demagnetization and recovery curves for this specimen were plotted it was apparent that they were somewhat different from those obtained from the steel specimens. The hysteresis seemed much larger. A part of this was due to the scale to which the curves were plotted, but when allowance was made for that, they still seemed to have an abnormally large area. This peculiarity of the curves for this specimen is undoubtedly due to eddy currents in the iron. These currents are in opposition to the time rate of change of magnetic flux and hence lag behind the magnetizing current by an amount somewhat in excess of 90°. They produce a magnetic field one component of which is in direct opposition to the applied field while the other reacts to cause a lag in phase of the field behind the current. Since the curves are plotted on the assumption that the applied field is proportional to the ordinate of the current curve, it is evident that the lag of induction behind the field intensity as given by the curves would be greater than the true hysteretic lag. When there is hysteresis without eddy currents the induction shows a lag behind the current at some points along the curve but none whatever at points of maximum or minimum field intensity. A difference of phase between the two curves on the photographs at their highest points is therefore evidence of the disturbing influence of eddy currents. Furthermore, if a method of correcting for eddy currents can be found, the phase relation of the highest points will furnish the criterion for determining the magnitude of the correction. Eddy currents are produced by the variation of the flux, ϕ, and their magnitude could be expressed as $K' (d\phi/dt)$, where K' is a negative constant. The current in the secondary, which operates one loop of the oscillograph, is also proportional to the derivative of the flux with reference to the time and the curves are marked $K (d\phi/dt)$ on the figures. Some fraction of $K (d\phi/dt)$ will equal $K' (d\phi/dt)$, in other words some fraction of the ordinate of the secondary curve on the oscillographic record will constitute the correction that should be applied to the corresponding ordinate of the current curve to give the ordinate that is proportional to the actual field intensity. It is comparatively easy to tell by inspection of the curves what this fraction is; or at least it may be found after two or three trials in applying the correction to the current curve. The flux and its time rate of variation are behind the current in phase at the highest points of the curves. If the fraction chosen be too small, the flux will be behind the corrected curve while if the fraction chosen be too large the flux curve will fall ahead of the new curve in phase at the highest points. A value of the fraction is chosen which places the corrected curve in phase at the highest point with the flux curve and then the ordinates of this new curve represent the resultant or actual field intensity applied to the iron. Fig. 9 shows the corrected curve with points indicated by the small circles. The correction applied to each ordinate of the curve was 13 per cent. of the corresponding ordinate of the secondary curve. The hysteresis for this specimen, as shown by the corrected curves, is found to differ very little from that shown by the other specimens for an equal change of flux.

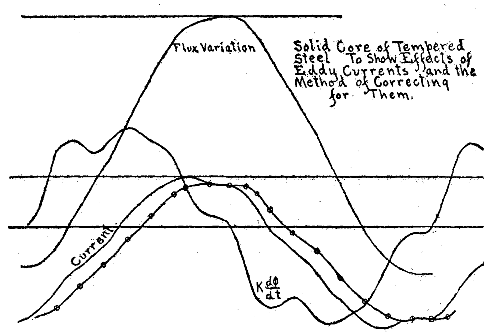


Fig. 9.

In order to further test this method of correcting for eddy currents, a solid magnet of tempered steel was used. Eddy currents would certainly be large in this case, but it was arranged to reduce end effects to a negligible magnitude by filling the small air gap with a carefully fitted block of iron. Fig. 10 shows the results. The curve plotted from the corrected data shows no more hysteresis than was found for the wire ring.

To obtain further data on the relative magnitude of the hysteresis in steel and soft iron for the kind of cycle here considered, a ring of soft iron wire was used. Wire having a diameter of .96 mm. was chosen. The wire of specimen *A* had a diameter 1.03 mm. If size of wire were all that determined the eddy currents effect, we should be assured of less disturbance from them with this new ring than was experienced with specimen *A*. However, in getting the total hysteresis curve this is not the case. With soft iron the hysteresis curve is nearly parallel to the axis of ordinates for a long way and hence the time rate of change of flux along this part of the curve is much greater than it is for hard steel where the slope of the curve is much less, hence in plotting the total hysteresis curve of soft iron, we should expect greater errors due to eddy currents than would arise in the case of hard steel. This argument, however, does not apply to demagnetization and recovery curves since the slope does not vary greatly from point to point along the curve and the time required for demagnetization or recovery is 1/120 of a second irrespective of the magnitude of the change. Hence for a given flux variation and for a given size of wire, effects due to eddy currents for soft iron ought not to differ widely from those in steel. Curve *D* of Fig. 12 was obtained from this new ring of soft iron wire. The phase difference at the maximum and minimum points of the current and flux curves was so slight that no attempt was made to correct for eddy currents. Apparently the hysteresis is somewhat larger than for the steel

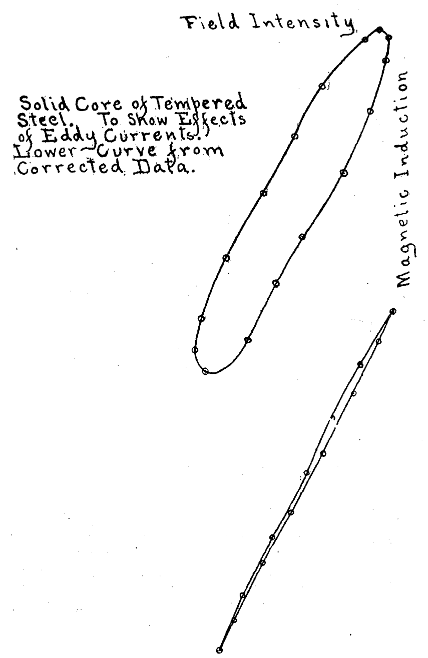


Fig. 10.

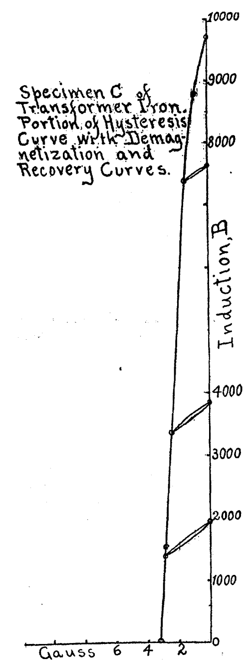


Fig. 11.

specimens. The conclusion is certainly justified that for cycles of the kind here considered the hysteresis of very hard steel is not greater than that of soft iron for an equal variation of flux density and it may be somewhat less.

The hysteresis curve for ring *C* was plotted ballistically in Fig. 11 and the recovery curves drawn to the same scale as the hysteresis curve are also shown in their relation to the residual magnetic induction. The recovery in the region of small residual magnetism in this case is only about 6 per cent. of the maximum residual induction.

Fig. 12 is for the purpose of comparing the slopes of the curves for the different specimens.

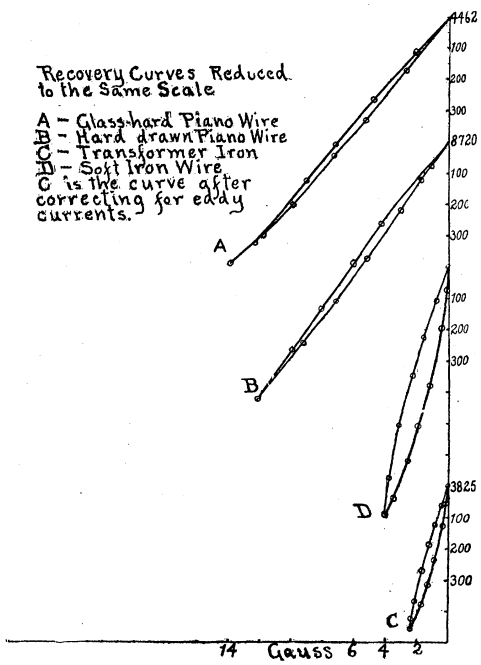


Fig. 12.

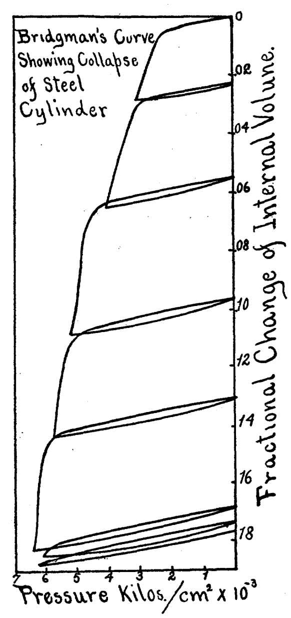


Fig. 13.

C. CONCLUSIONS.

1. An oscillographic method has been developed for plotting any kind of a magnetic cycle with an alternating current of 60 cycles provided eddy currents and end effects are negligible. The method is applicable when the flux variation is as small as 150 lines of induction and it may be used for any cycle in which the flux variation is greater than that amount.

2. It was shown that in a given specimen, for any value of residual magnetism, there is a definite elastic limit which is the maximum field that can be applied to the specimen without producing permanent demagnetization; and that if any field not exceeding the limiting field be applied and then removed, the specimen will return to its initial condition.

3. Within the elastic limit, magnetic stress and strain are approximately proportional, *i. e.*, the demagnetization is approximately proportional to the field intensity that produces it, provided the specimen returns to its original remanence when the field is reduced to zero.

4. The hysteresis as indicated by the area of the curves is small and is approximately the same for soft iron and hard steel.

5. The extreme points of the stable demagnetization and recovery curves lie inside the maximum hysteresis curve.

6. The susceptibility to small demagnetizing fields increases slightly with decrease of residual magnetism, the maximum difference observed being about eight per cent.

7. The recovery from the effect of a demagnetizing field is greater for low value of residual induction. For hardened steel it was found to be 1,960 lines of induction per sq. cm. or 24.5 per cent. of the maximum residual induction. For drawn piano wire it was 1,200 lines per sq. cm. or 8 per cent. of the maximum remanence and for soft iron it was 600 lines per sq. cm. or 6 per cent. of the maximum remanence. These numbers represent the elastic limits for low values of residual magnetism.

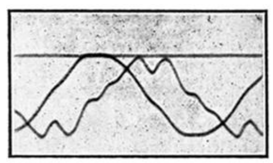
8. There is shown to be a well-defined upper limit beyond which the residual magnetism of an open magnetic circuit of steel can not be made to pass.

9. The slope of the demagnetization curves for hard steel is 57. For softer specimens it is larger and it reaches a value for very soft iron of about 180.

10. For hard steel the elastic limit is approximately proportional to the difference between the residual magnetism and the maximum residual magnetism.

The work was done in the physical laboratory of the University of Michigan under the direction of Professor K. E. Guthe. The author wishes to express his appreciation of Professor Guthe's interest in the work and his helpful suggestions while it was in progress.

University of Michigan,
Ann Arbor, June 1, 1912.



Oscillogram. The smooth curve represents the unidirectional primary current; the irregular curve the secondary current.

Fig. 2*a*.